

A LEAST ERROR SQUARES TECHNIQUE FOR ESTIMATING
THE MAGNITUDE AND FREQUENCY OF A VOLTAGE SIGNAL

A Thesis

Submitted to the Faculty of Graduate Studies and Research
in Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy

in the Department of Electrical Engineering
University of Saskatchewan

by

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Saskatoon, Saskatchewan
October 1986

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ACKNOWLEDGMENTS

The author would like to thank Dr. M. S. Sachdev for his guidance throughout the course of his work.

Special thanks are extended to his wife, Pinar and his daughter, Elif, for their encouragement and patience during the study.

Financial assistance provided by the Natural Sciences and Engineering Research Council of Canada in the form of a research assistantship is gratefully acknowledged.

UNIVERSITY OF SASKATCHEWAN

Electrical Engineering Abstract 86A267

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Ph.D. Thesis Presented to the
College of Graduate Studies and Research

October 1986

ABSTRACT

The use of digital processors in implementing protective relaying functions has attracted considerable attention in the past several years. A major part of a digital processor based relay is the algorithm which estimates the system parameters such as voltage, current and frequency, using the digitized samples obtained from the system. Developments of suitable algorithms to detect these parameters have been a major challenge to many researchers.

This project is concerned with the development of an algorithm for estimating the magnitude and frequency of a voltage signal using the least error squares curve fitting technique. One of the main considerations is the validity of the algorithm when the magnitude and/or the frequency of the voltage substantially deviate from their nominal values. The effective use of the algorithm under such conditions has been accomplished. The use of the proposed algorithm for implementing a software based volts-per-hertz relay is also demonstrated in this project.

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