بسم الله الرحمن الرحيم

In the name of Allah, the Beneficent, the Merciful

ANALYSIS AND STABILITY OF A CAPACITOR COMPENSATED RELUCTANCE MOTOR

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by
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"ANALYSIS AND STABILITY OF A CAPACITOR COMPENSATED RELUCTANCE MOTOR"

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ABSTRACT

With the development of modern solid-state power supplies, there has been an increasing demand for variable speed synchronous drives. In several applications, a high degree of rotational stability and uniformity of speed is required. Reluctance motors are considered to be suitable electro-mechanical drives for such purposes.

A general method of analysis based on the physical understanding of the motor is presented in the thesis. The analysis is developed for a three-phase capacitor comepnsated reluctance motor under asynchronous operation. The method yields the voltage equations, equivalent circuits, expressions for the various currents and expressions for the torques including the pulsating torque. The total performance of the motor is discussed in detail. The experimental results of an especially designed reluctance motor confirm the validity of the analysis.

To study the stability behaviour of the motor, the damping and spring coefficients are investigated in detail for various modes of operation.

The investigations presented in the thesis reveal that a variable speed reluctance motor with a very high degree of stability and uniformity of speed can be designed. Some topics which require further investigations are outlined.

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TABLE OF CONTENTS

LIST OF FIGURES

Figure Page

Figure Page

LIST OF SYMBOLS

Symbol	Parameter
$a_{(x,t)}$	Current-loading with respect to space and time
$b_{(x,t)}$	Air-gap flux density with respect to space and time
C	Capacitance of the compensating capacitor
c	Spring coefficient
d	Damping coefficient
E_1	Internal voltage of the power supply
f, ω	Supply frequency
g_1, g_2	Effective air-gap length under the pole, in the interpolar region, respectively
$\dot{I}'_1$, $\dot{I}'_3$	Stator phase current of frequency ω and $(2s-1)\omega$, respectively
$\dot{I}''_p$, $\dot{I}''_n$	Positive and negative sequence currents in rotor bars
$\dot{I}_{N1}$, $\dot{I}_{N3}$	Currents in the power supply
$\dot{I}_{c1}, \dot{I}_{c3}$	Capacitor currents
k_1	Constant representing the rotor electrical asymmetry
k_2	Constant representing the magnetic asymmetry
L'_{11}	Useful self rotating inductance of a stator phase independent of rotor position
L''_{22}	Useful self rotating inductance of a rotor mesh
L'_σ	Leakage inductance of a stator phase
L''_σ	Effective leakage inductance of a rotor mesh
ℓ	Length of rotor laminations
$M_{12} = M_{21}$	Maximum value of the mutual inductance between a stator phase and an ideal rotor mesh

N_{ph}	Number of turns of a stator phase
p	Number of pairs of poles
R	Bore radius
R'	Stator phase resistance which includes any external series resistance
R''	Effective resistance of a rotor mesh
R_N	Internal resistance of the power supply
s	Slip
T	Total instantaneous electromagnetic torque
T_{as}	Electromagnetic asynchronous torque
T_{pul}	Electromagnetic pulsating torque
t	Time, seconds
$U_{(x,t)}$	MMF with respect to space and time
V_1	Phase voltage at the motor terminals
x'	Co-ordinate of a point measured from the magnetic axis of a stator phase, electric radians
x''	Co-ordinate of a point measured from the pole axis of the rotor, electric radians
Z''	Number of rotor bars per pole
α	Electrical angle between two consecutive bars
β	Pole arc to pole pitch ratio
ϕ	Corresponding phase angle
Φ_p	Electrical angle between the rotor pole axis and the magnetic axis of stator phase at $t = 0$
δ	Load angle, electrical
τ_p	Pole pitch
ω_r	Rotor speed, electrical rad/s
λ	Permeance per unit area
ξ_1	Winding factor of a stator phase
μ_o	Permeability of free space

Regarding super-scripts, single prime (') refers to the stator quantities and double prime (") refers to the rotor quantities. The subscripts (o) indicates the steady-state value of a quantity at s=0 and $\delta=\delta_o$.

1. INTRODUCTION

1.1 General Review

In recent years, there has been an increasing demand for variable speed synchronous drives. Such drives are used in many applications, for example, in metered pumping in the chemical industry, in high speed cutting and grinding, in fibre processing, in precise and reliable positioning of the control rods of nuclear reactors, etc.

An extremely high degree of rotational stability and an absolute uniformity of speed is required in several applications. As an example, in the drive system of a photo-transmission equipment, any torsional oscillations are highly undesirable. Further, the electro-mechanical drive is expected to be both self-starting and self-synchronizing.

Permasyn motors (permanent magnet synchronous motors) and hysteresis motors have been used for such purposes because of some of their outstanding advantages. Apart from their complicated rotor construction, which requires the use of special kinds of hard magnetic materials, both motors possess some specific disadvantages. With regard to the permasyn motor, the investigations carried out on such a motor revealed that the motor, under certain circumstances, may not be self-starting and has an inherent tendency to oscillate.[1,2] On the other hand, the hysteresis motor, which involves the use of expensive alloy-steel, can only be built at reasonable costs in millihorsepower sizes. The cost of a hysteresis motor increases considerably with the increase in rated power of the motor.[3] Further, the efficiency and specific output of such a motor are relatively very low.[4] For these reasons, therefore, the reluctance motor can be considered as a more suitable electro-mechanical drive for such applications.

Despite the disadvantages of the reluctance motor of rather low power factor and specific output, it is becoming increasingly popular due to its outstanding advantages of cheap, robust and reliable construction. Moreover, a reluctance motor requires a lesser number of poles as compared to a synchronous motor for obtaining a specific resolution of the rotational speed under synchronous operation. Furthermore, a reluctance motor having a squirrel cage in its rotor has the capability of self-starting and self-synchronizing.

The reluctance motor is generally fed by a solid-state variable frequency power supply for variable speed synchronous operation. In general, such a supply possesses a relatively high internal resistance. A compensating capacitor is therefore provided at the motor terminals in order to reduce the large internal voltage drop of the supply. The value of the compensating capacitor plays an important role in the stability behaviour of the motor. In order to stabilize the motor, a capacitor of suitable value dpending on the power supply system has to be chosen.

1.2 Comments on Published Literature

Although a considerable amount of information about reluctance motors is available in the published literature, most of the publications deal with their synchronous operation.[5-22] The information about the asynchronous operation of reluctance motors is rather limited and insufficient.[23-26]

Although one of the publications describes a very useful approach for the determination of the asynchronous operation of a reluctance motor, the analysis is limited to a motor without damper bars.[27] Further, the pulsating torque, which gives substantial information about the smooth operation both at starting and running conditions, has not received much attention. In addition, with the exception of Lin's paper,[8] an equivalent circuit of the reluctance motor with damper bars under asynchronous operation is not available.[28] It may be mentioned that the equivalent circuit described by Lin is quite complicated.

Some interesting papers on the stability of reluctance motors have been published recently.[29-33] However, the information about electromagnetic damping and spring coefficients, which characterize the stability behaviour of the motor, is extremely limited in the published work.

It may be added that the case of a capacitor compensated reluctance motor fed by a variable frequency power supply having a large internal resistance represents a typical mode of operation in several practical applications. Hardly any information about the asynchronous and stability performance of reluctance motors under such a mode of operation is available.

It is felt that there is a considerable need for the development of a method of analysis of a general nature, which incorporates a direct and simple approach based on the physical understanding of the problem, for the investigation of asynchronous operation of a compensated reluctance motor. The analysis thus obtained could then be extended for the study of stability behaviour of the motor under various modes of operation.

1.3 The Problem

In order to develop and design a highly stable variable speed synchronous reluctance motor with an extremely high uniformity of speed, one requires complete information about the total performance of the reluctance motor under various modes of operation. The object of this thesis is to develop a suitable method of analysis for a three phase reluctance motor with damper bars taking into account the effect of supply internal resistance and compensating capacitor. The method of analysis should deliver expressions for various currents and torques, including pulsating torque, considering both electrical and magnetic asymmetries. For the investigations of stability, expressions for the electromagnetic damping and spring coefficients are required.

1.4 Brief Description of Contents

The thesis consists of mainly two sections. The first section deals with the analysis of asynchronous operation of a capacitor compensated reluctance motor having damper bars in the rotor. The second section contains the stability analysis of such a motor.

After considering the nature of various currents in the stator and rotor, an analysis based on the physical understanding of the reluctance motor is developed. An attempt has been made to take the effect of the rotor bars into consideration in a fairly simple way, which is considerably different from the various approaches adopted in the published literature. The method presented here simplifies the determination of the distribution of rotor bar currents to a great extent.

By using the MMF method, the voltage equations are developed for the asynchronous operation. The expressions for various currents and torques are then obtained. By suitably modifying the voltage equations, an equivalent circuit of the capacitor compensated reluctance motor is derived.

In order to obtain the total asynchronous performance of the motor, the operation is considered under two conditions of the power supply. In the first case, the internal voltage of the power supply is assumed to remain at a constant value, while in the second case, the voltage at the terminals of the motor is considered to be constant. The variation of the various currents and torques is obtained for different modes of operation and is discussed in detail.

The stability analysis in synchronous operation is developed by considering a slight deviation from the synchronous speed in the event of any disturbance. The method of investigating stability is considerably simplified by assuming the frequency of rotor oscillations to be very small in comparison with the supply frequency, which is usually the case in such drive-systems. The expressions of electromagnetic damping and spring coefficients are derived by expanding the torque equations in the neighbourhood of an operating point at synchronous speed and a specified load angle. The characteristics of damping and spring coefficients are then obtained and discussed for various modes of operation.

Five appendices are given at the end of this thesis. Appendix-1 describes the derivation of the MMF distribution of rotor bars. The permeance equation of the motor is considered in Appendix-2. Appendix-3 presents the derivation of the effective parameters of a rotor mesh, and Appendix-4 gives the derivation of the torque equations, whereas in Appendix-5, the relationship between the pole-wheel angle and the load angle is explained.

2. PHYSICAL INTERPRETATION OF RELUCTANCE MOTOR UNDER ASYNCHRONOUS OPERATION

Before proceeding with the mathematical analysis of the motor under investigation, a physical understanding of the working of the motor under asynchronous operation is essential. By considering the magnetic and electrical arrangements of the motor, the nature of various currents in both stator and rotor can be found. The behaviour of the motor at starting and in synchronism can then be found by considering particular cases of the asynchronous operation.

2.1 Construction of Reluctance Motor

The reluctance motor is essentially a synchronous machine without any field excitation. The stator carries a polyphase winding, which is similar to that of an ordinary synchronous or induction machine. The rotor is, basically, a squirrel-cage induction machine rotor. In order to have the saliency in its magnetic circuit, the rotor is shaped either by the use of special punching or by milling away portions of a normal rotor. Further, as the case considered in this thesis, the rotor bars may not be uniformly distributed around the rotor circumference. It may be mentioned that there could be several configurations for the rotor of a reluctance machine so as to improve the motor performance. However, the conventional rotor is considered in the thesis. Fig. 2.1 shows a simple cross-sectional view of a 4-pole reluctance motor. The machine to be analysed hereafter has a construction similar to that shown in Fig. 2.1.

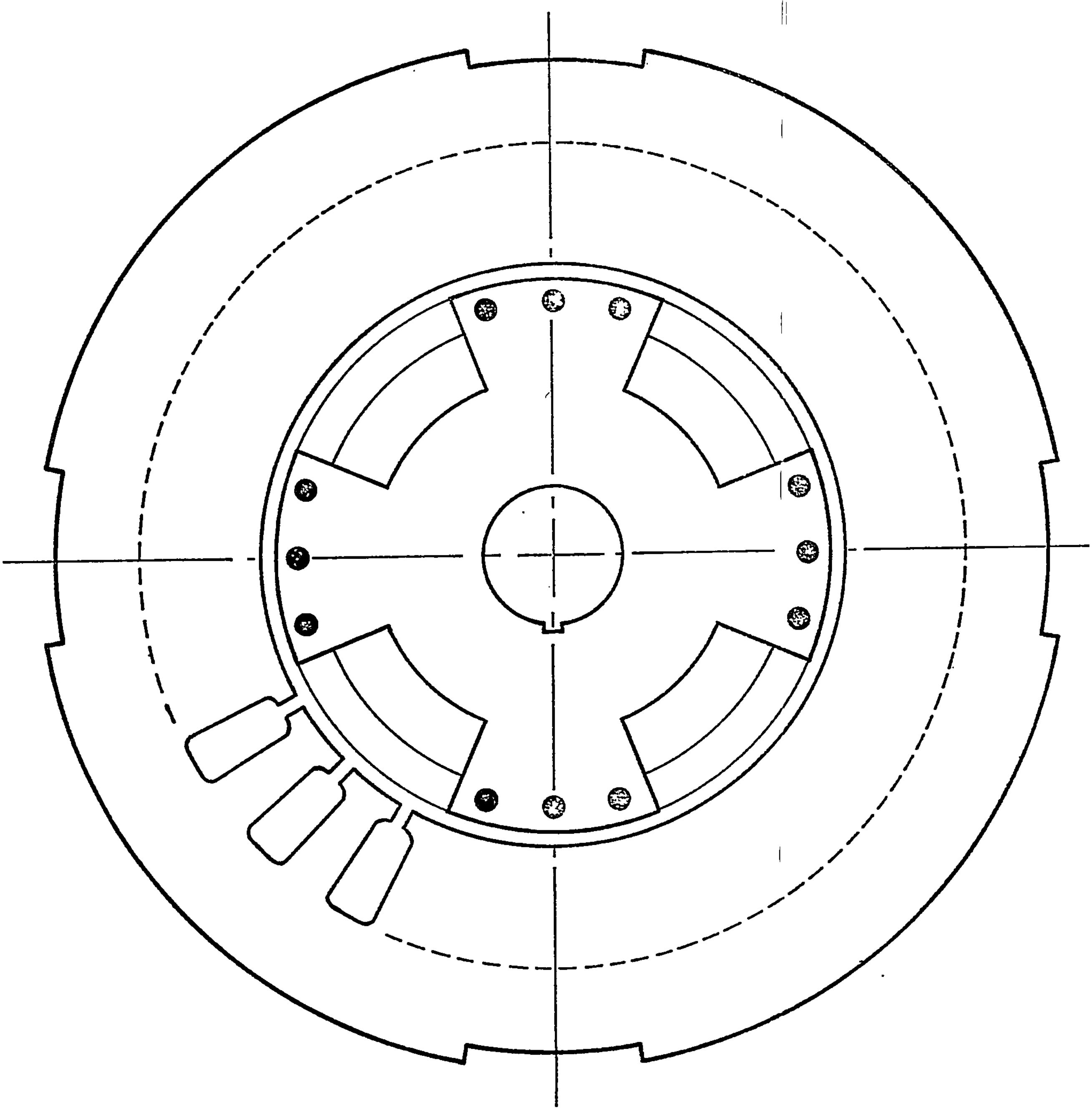

Fig. 2.1 Cross section of a 4-pole reluctance motor.

From Fig. 2.1, it is evident that the motor possesses two asymmetries, namely

a) magnetic asymmetry, which is due to the inequality of the air-gap length under the pole and in the interpole region, and

b) electrical asymmetry which is due to the non-uniform distribution of the damper bars around the periphery of the rotor.

In the analysis presented here, each asymmetry is expressed through a constant depending upon the configuration parameters of the machine. This representation is very useful in studying the effect of the degree of each asymmetry on the performance of such a machine.

The purpose of the bars provided in the rotor is to act as a starting device because they provide the required induction motor torque, which brings the rotor up to a speed from which it can synchronize with the stator magnetic field. Another function of the rotor bars is to act as a damping device for damping the oscillations in synchronous operation. As the rotor bars are inactive under absolute synchronous operation, the synchronous torque must be produced due to the reluctance variation of the air-gap.

After having considered the machine construction, the nature of the various currents of the motor under asynchronous operation will now be discussed.

2.2 Nature of Currents

2.2.1 Stator currents

The supply voltage of frequency ω produces a stator current i_1' of the same frequency, which is supposed to have a positive sequence. Due to the magnetic asymmetry of rotor, however, a rotating field component of negative sequence is produced in addition to the main positive sequence rotating field. This negative sequence component possesses a frequency of $(2s-1)\omega$.[26] The rotational slip s is defined as:

$$s = \frac{\omega - \omega_r}{\omega} .$$

The negative sequence field component induces a voltage of frequency $(2s-1)\omega$, to which the power supply acts like a short circuit, which produces a negative sequence current i_3' of the same frequency in the stator.[34]

2.2.2 Rotor currents

The currents i_p'' of positive sequence and frequency $s\omega$ are expected in the rotor bars. Due to the electrical asymmetry, negative sequence currents also of frequency $s\omega$ are present in the bars.[35] At the same time, the stator negative sequence field component caused by the magnetic asymmetry also produces currents of negative sequence and frequency $s\omega$.

Thus, by taking the combined effects of both electrical and magnetic asymmetries into consideration, the currents i_p'' and i_n'' will be present in the rotor bars, and i_1' and i_3' will be present in the stator winding under asynchronous operation. Obviously, the magnitude of these currents depends on the rotational slip and the degree of asymmetries.

The various currents in the stator and rotor are shown in Fig. 2.2.

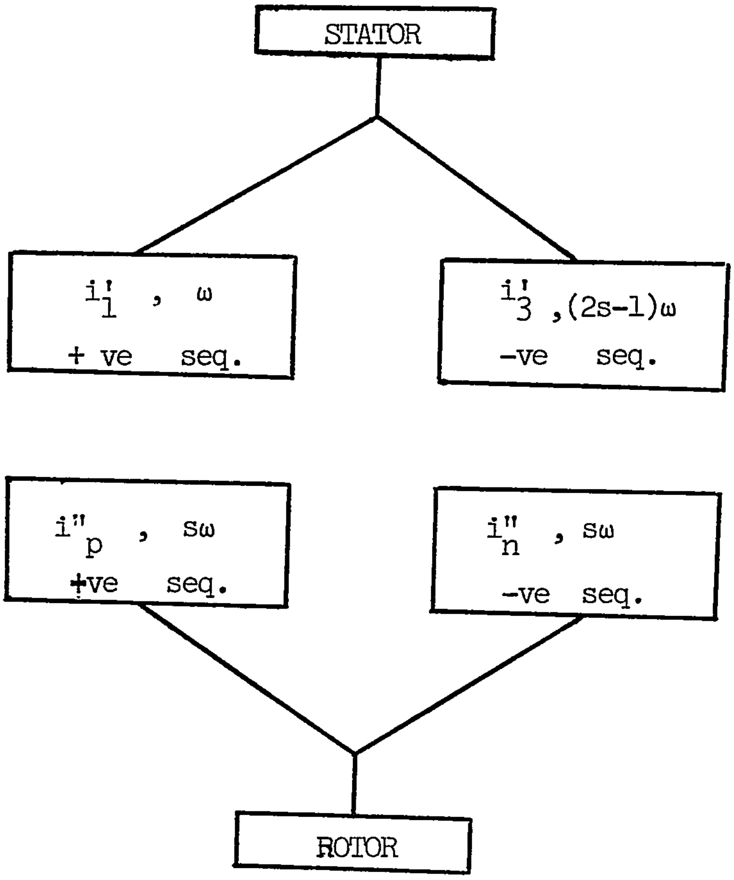

Fig. 2.2 Nature of currents in stator and rotor.

After knowing the nature of currents, the magnetic field components in the air-gap can then be readily described.

2.3 Air-Gap Field Components

In a normal symmetrical polyphase stator winding, a positive sequence current of a certain frequency produces a rotating positive sequence MMF having the same electrical frequency of rotation. It is worthwhile to add here that both MMF and current-loading have the same nature. Due to the presence of a variable air-gap, i.e., magnetic asymmetry, the positive rotating MMF would give rise to two magnetic field components of opposite sequence and unequal magnitude in the air-gap. However, it should be noted that the frequency of rotation of the negative sequence field component thus produced is (2s-1) times the frequency of rotation of the positive sequence field.

Hence, with the presence of both currents, namely i'_1 of ω and i'_3 of $(2s-1)\omega$, in the stator, four magnetic field components will be present in the air-gap.

Now considering the rotor, the positive sequence rotor currents i''_p of $s\omega$ frequency produce two rotating MMFs of opposite sequence due to the electrical asymmetry of the rotor bars. Again, due to the magnetic asymmetry, each MMF would give rise to two magnetic field components of opposite sequence. Hence, the positive sequence currents i''_p will produce four field components. Similarly, the negative sequence currents i''_n will also produce four field components. Regardless of the sequence, all the eight field components so produced have the frequency of rotation of $s\omega$ with respect to the rotor. For the sake of convenience, all these field components produced by the rotor are described with reference to the stator. Hence, the frequency of rotation of the positive sequence rotor field components will be ω, whereas that of the negative sequence rotor field components will be (2s-1)ω.

Having obtained the nature of currents and magnetic field components in the air-gap, the torque produced by the motor can then be discussed.

2.4 Torque

2.4.1 General

For the calculation of the instantaneous value of torque, one usually makes use of Biot-Savart's law. This requires the knowledge of the various magnetic field components in the air-gap and the distribution of the reacting currents which are usually treated as current-sheets and expressed in terms of current-loading components.

In order to calculate the total torque produced by a machine having magnetic asymmetry, it is advisable to consider the interaction between the resultant air-gap field and the current-loading due to the stator conductors.[26,27] Both magnetic field and current-loading are described by their rotating **Fourier** components in order to simplify the torque calculations.

The nature of a torque produced depends on the relative displacement between the magnetic field component and the current-loading component under consideration.[36]

The various torques can be treated under two different categories.

2.4.2 Time independent torque

The torque produced by a current-loading component reacting with a magnetic field component would be time independent if the relative displacement between them is also time independent. This condition will be fulfilled if the speed of the two components under consideration is the same. If this condition is satisfied at any rotor speed, the torque thus produced is called an Asynchronous Torque.

The various components of current-loading and magnetic field are shown in Fig. 2.3. The asynchronous torque components are found to be due to the interaction between:

a) a'_1 and b'_{31}, b''_{p11}, b''_{p31}, b''_{n11} & b''_{n31}

and b) a'_3 and b'_{12}, b''_{p12}, b''_{p32}, b''_{n12} & b''_{n32}.

Although the condition for producing a time independent torque (i.e., the speed of the two components under consideration is the same) is fulfilled by the pairs a'_1 & b'_{11} and a'_3 & b'_{32} they produce a zero torque separately due to the fact that both components of a pair have the same phase angle.

It may be added that the sense of the torque is in the same direction as that of the rotation of the interacting components.

2.4.3 Time dependent torque

On the other hand, if the current-loading and the magnetic field rotating component are not relatively at rest, the torque produced will be time dependent. As the speed of each component is