TRANSMISSION SYSTEM EXPANSION PLANNING UNDER DEREGULATION

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by

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ABSTRACT

One of the main objectives of deregulation in electric power industry is to introduce competition in the generation sector. The introduction of competition in electric power market has made power flow control more complex and difficult than ever. The scarcity of transmission capacity and a continued demand for power from less expensive sources sometimes lead to transmission system congestion or overloading. Congestion relief through either generation reschedules or transmission system expansion becomes an economic issue on which the deregulated electric industry is showing an increasing interest.

The research work described in this thesis examines issues related to transmission system expansion under deregulation, including transmission congestion, determination of optimal generation reschedules and selection of optimal transmission system expansion schemes.

When congestion occurs in a deregulated power system, generation has to be rescheduled to ensure system security. This reschedule usually increases the operating cost. Two methods, named DC and AC method respectively, are proposed in this thesis to optimize the generation reschedule.

An optimal congestion fee can be assessed based on the optimal generation reschedule. Transmission system should be expanded if the congestion fee is higher

than the investment cost of building a new transmission line to relieve the congestion. This thesis reports a detailed analysis on this issue. A calculation process of determining optimal transmission expansion scheme is presented as well.

The short-term power market has a great impact on transmission system expansion due to its uncertainty that only makes the long-term planning of system expansion more complicated. This influence is also discussed in this thesis and a method is proposed to include the effects of short-term power market into the overall solution of transmission system expansion.

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CHAPTER 1: INTRODUCTION

1.1 Introduction to Power System

One of the most significant features of our present-day civilization is the extensive use of electrical energy to meet the increasing requirements of our daily activities. Electro-mechanical energy conversion permits the generation of electrical energy to be concentrated in large, efficient stations at favourable locations and makes possible the large-scale utilization of hydraulic power. Electrical energy can be transmitted over long distances with good efficiency, economy and reliability. It can be readily converted at the place of utilization to a desired form of power and thus can produce light, heat, chemical action and mechanical power. Electric power is clean, convenient and can be easily controlled in its usage compared to other forms of energy.

Power system is a composite entity made up of three sub-systems: generation, transmission and distribution. Operating a power system is a complicated task that is not only concerned with quality, system security and reliability, but also concerned with environmental and regulatory requirements.

A typical structure of a power system is shown in Fig. 1.1. Electric power is generated at power plants. It is delivered through the transmission system to the customers who are the users of electrical energy. The customers are usually served by a web of networks termed as a distribution system. Unlike other forms of energy such as fossil oil, coal, etc, the electrical energy cannot be stored in large amount. This nature of electrical energy dictates that the entire process of electrical power production, transmission and consumption must happen at the same time. The major characteristics of a power system are that, at any time, the electrical energy produced by its generators must be equal to the electrical energy consumed by its customers plus the transmission loss. In other words, the electrical energy must be kept balanced at any instant.

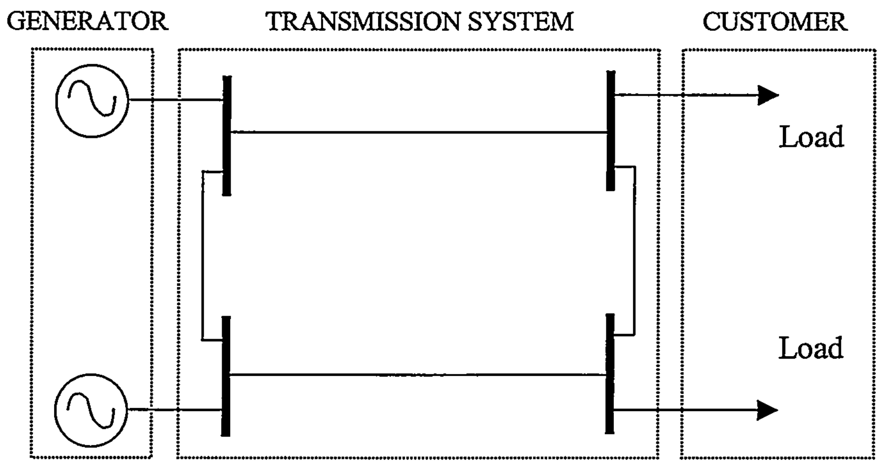


Fig. 1.1 A Typical Structure of a Power System

The first commercial power system in the world was constructed and placed in operation in 1882 by Thomas Alva Edison in New York City [1]. It was a direct current system that served a limited number of consumers in the vicinity of the plant at a nominal voltage of 100 volts. The invention of transformer in 1883 in England by John Gibbs and Lucien Gaullard, together with the invention of the alternating current induction motor and the development of polyphase circuitry in 1886 by Professor Galileo Ferraris in Turin [2] opened the way for the adoption of alternating current and rapid expansion of electrical transmission systems.

A transmission system interconnects all generating stations and load centres in a power system and forms the backbone of an integrated energy supply system. The evolution of transmission systems is characterised by progressive usage of higher voltage AC transmission, application of HVDC transmission technology and formation of large-scale interconnected electrical networks.

High voltage transmission can bring benefits to a power system. Transmission loss and voltage loss could be reduced greatly if a higher voltage were employed to transmit the same amount of electrical power. In other words, the transmission capacity could be improved considerably in this case. With an increasing need for transmitting larger amount of electricity over longer distances, a modern transmission system is built with high voltage lines whose operating voltage could be several hundred kilovolts. 12, 44 and 60 kV was used in early AC transmission systems. Since then, transmission voltages have risen steadily, and the extra high voltage (EHV) in commercial use is 765 kV, which was first put into operation in the United States in 1969 [3].

HVDC transmission becomes more economical if bulk electrical power is transmitted over very long distances more than 500 km for overhead lines or 50 km for electrical cables [3]. DC lines have no reactance and are capable of transferring more power for the same conductor size than AC lines. HVDC transmission is especially advantageous when two remotely located large systems are to be connected. A HVDC transmission line acts as an asynchronous link between systems where AC interconnection would be impossible due to system stability consideration or different nominal frequencies used in these systems.

Power system security and economy of operation could be improved greatly with the interconnection of neighbouring electrical networks. These benefits come from the mutual emergency assistance and the need for less generating reserve capacity on each system. Also, interconnection makes the electrical energy generation and transfer more economical since power can be readily transmitted from one area to others. Sometimes, it may be cheaper for a system to buy bulk power from neighbouring systems than to produce it in one of its own plants. These benefits encourage the formation of large-scale transmission systems spanning over larger areas.

1.2 Regulation and Deregulation of Power Systems

For nearly a hundred years, from the late 19th century to the end of the 20th, a power system operated as a regulated monopoly. Strictly speaking, power system regulation means that the government has set down laws and rules that put limits on and define how the electrical industry or company can operate. Under regulation, a power company is granted monopoly franchise by the government to sell electricity to consumers in a certain area, called the franchise territory. Within this service territory, no other company can produce and commercially sell electrical power. In a regulated monopoly, a power company has obligation to provide service to its customers with regulated rate and limited benefit return. The government may put strict limitations on what and how the local power company functions. These could be anything from a requirement on how it builds its system, or the way it does its planning, to strict definitions on how it finances its operations. In addition, the government will define how the company computes costs and sets its prices. Usually, it requires that the company operate the electrical system in a lowest cost basis. As shown in Fig. 1.2, a traditional regulated power company usually integrates all the electrical services: generation, transmission, distribution and retail sales. The advantage of a regulated power industry is that electrical systems can be controlled and managed centrally. In addition, regulation guarantees the investors a fair return and reduces the risk of investment, and encourages utilities to invest large amount of capital to expand their electrical networks. Actually, power industry regulation has made a great contribution to the formation of large-scale modern power systems.

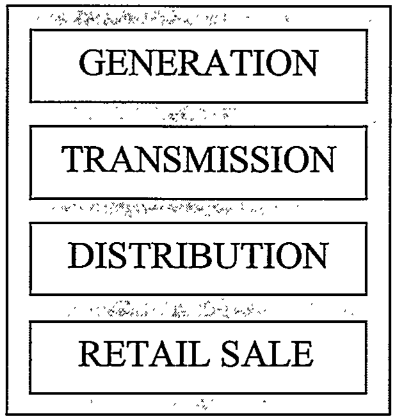


Fig. 1.2 Traditional Regulated Utilities Model

Deregulation of power industry has become a growing trend all over the world since 1980's when Chile became the first country to make such a re-organization [4]. Basically, the drive for electrical industry deregulation came from the Chilean government's decision to introduce competition among electricity suppliers and let customers have wide choices to buy electrical power from different sources. Customer is defined as bulk electricity consumer or electricity retailer in deregulated electrical industry. Governments of many countries around the world value the advantages brought by deregulation more than the benefits of continuing utility regulation. These advantages are:

- Competition among power suppliers can provide incentives for innovation and improve efficiency which will eventually bring down the generation cost and electricity price.

- Competition will also promote customer focus on the part of suppliers. It means that power company will anticipate their customers' needs and respond to them in a competitive environment.
- Deregulation will provide customers with more choices that are not only the price of electric energy, but also electrical services of high quality such as the supply of firm generating capacity.

Under deregulation, a previously integrated power company will decompose itself by function into separate companies, which may still be owned by the same stockholders, but will be legally separate entities. Each part will work independently. In many parts of the world including North America, generation section and retail sale section are deregulated. Therefore, generation companies and power retailers compete in a competitive power market. Transmission and distribution systems are still regulated to provide open and fair access to power systems for all power producers and buyers. Customers can choose to buy electricity from power producers directly or purchase it from retailers. This restructure is shown in Fig. 1.3.

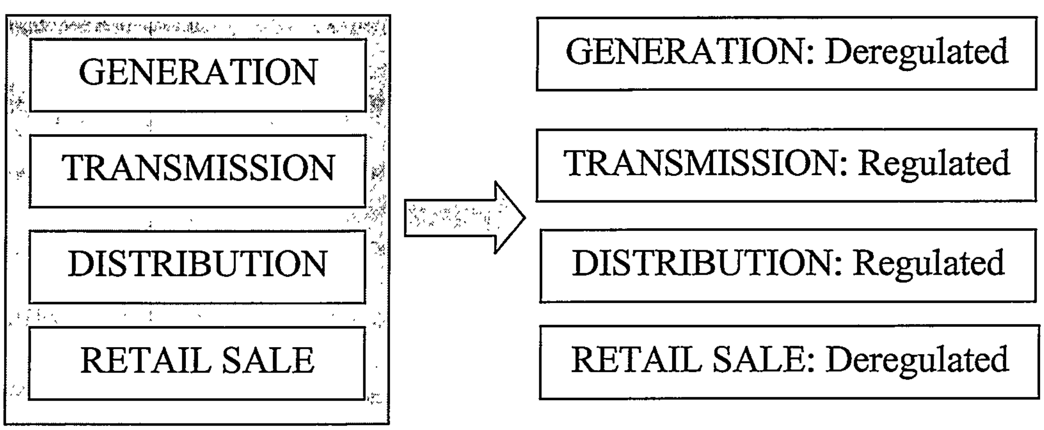


Fig. 1.3 Deregulated Utilities Model

Although the basic function of a power system is not changed by deregulation, the structure of electrical industry has been changed in a fundamental way. Deregulation has brought forward many new challenges that have to be addressed. New planning, operating and marketing issues related to deregulation are evolving and these issues are being addressed.

The implementation of deregulation requires the establishment of an Independent System Operator (ISO) to manage and control the system operation and ensure the transmission system open to all participants including generation companies and customers equally. ISO is an impartial organization that has no interest on electrical power trade. Meanwhile, deregulation will also need the establishment of power exchange market where participants can exchange power freely in terms of the principle of discriminative or non-discriminative auction. Of course, power sellers and buyers can also make private deals with each other under deregulation.

The problems and many challenges caused by deregulation have gradually drawn the attention of researchers since 1980's. Prior to that, investigation on power system issues such as economic operation, generating reserve and power system expansion were all based on centralized monopoly system. With the implementation of deregulation policy in an increasing number of countries and regions, researchers have become more and more interested in different aspects of deregulation.

Although a transmission system is still subject to regulation in a deregulated system, its operating philosophy is different from that of a traditional transmission system. One of the major changes brought by deregulation is that generators are not required to be controlled by system operators. Generation pattern is completely dependent upon market. In this case, system power flow control, however, becomes more complicated than that in a regulated system. Market based power flow eventually results in transmission congestion. Transmission congestion must be eliminated to ensure the security of system operation. System operators usually have two ways to relieve congestion: rescheduling generation in terms of optimal cost principle or expanding the transmission system. This thesis mainly focuses on investigating two issues: transmission congestion and how to expand the existing transmission system if congestion occurs.

1.3 Literature Survey

Garver is the first researcher to formulate transmission expansion problem with the help of mathematical model in the early seventies [5]. Garver's mathematical model is oriented and applicable to the regulated power industry. Since then, many new methods and techniques about optimizing transmission expansion have been developed on the basis of Garver's model. This model requires that the transmission system planning should be performed by vertically integrated utilities with strict regulatory review of transmission capacity additions. Generally speaking, traditional optimal methodology in transmission expansion mainly focuses on long-term transmission network planning. The optimal objective is to search for those appropriate lines from the candidate circuits to minimize the investment and operational costs, and at the same time, to meet the forecasted load demand along the planning horizon.

Transmission expansion is usually a problem of non-convexity that is hard to obtain global solutions for complex, actual network. Among the optimization techniques utilized, Benders decomposition [6] has been used frequently by researchers. In the

Benders approach, the original network design problem can be decomposed into two subproblems: the master and slave subproblems. The master subproblem models only investment variables and proposes network expansion plan. The slave subproblem implements the expansion plans suggested by master subproblem and checks its network feasibility. The iteration between both subproblems is characterized by Benders cuts, which are evaluated from the slave's solution and added to the master subproblem. Usually, a mixed, non-linear, non-convex formulation is used to formulate the power transmission network expansion problems. To handle with this non-convexity difficulty, a Bender hierarchical decomposition approach was proposed by Romero and Monticelli [7]. This approach utilizes three different levels of network modeling: transportation models, hybrid models and linearized power flow models (DC models). Each level of network representation is seen as a relaxation of the next higher level. The first two models relax all the non-convexities constraints, which results in the initial optimal solution. Then, the non-convexities are introduced (third model). The key point of this approach is that when the method eventually switches to the unrelaxed network model with the initial optimal solution, it will be sufficiently close to the global optimal solution and the chance of reaching a local optimum is greatly reduced. Practical experiences showed very good results of this approach with a real world case study [8]. Following this work, Oliveira, Costa and Binato [9] have proposed a two-phase Benders hierarchical model but using the Geoffrion heuristic approach [10] within the Branch-and-Bound algorithm to solve the master or investment subproblem. Nevertheless, even with good results, it is not possible to guarantee the optimality of the solution to be found because the non-convexities still exist in the mathematical model used.

The main drawback of Benders decomposition approach is that it easily converges to a local optimal solution due to the non-convexities of transmission expansion problem. This situation has prompted the researchers seek other ways to solve the problem. Some other optimization techniques used by researchers recently are: Genetic Algorithm [11], Simulated Annealing [12] and Tabu Search [13]. Genetic algorithm is an optimization method that works above a set of candidate solutions and performs a number of operations based on genetic mechanism. This method uses a selection mechanism in terms of its main objective to select out some good solutions from the current ones, called parents, and then, recombine the information contained in them to create new solutions, called sons. The whole process repeats until the optimal solution is found. Simulated Annealing is another optimization technique that simulates the annealing process of molten material and is based on the fact that nature performs an optimization of the total energy of a crystalline solid when it is annealed under a very slow cooling schedule. Mathematically, this optimization process starts from a random point in a search space and a random move is made. If the new point is better than the previous one, it is accepted, otherwise, it is accepted only with some probability that depends on a certain temperature. This temperature starts at a high value and is slowly reduced according to a cooling schedule. Tabu Search is an iterative search procedure that moves from one solution to another to look for improvements on the best solution visited. A movement is an operation to jump from one solution to another while memory is used with different objectives such as to guide the search to avoid cycles. Using the concept of memory, specific movements are forbidden.

Since electrical deregulation began in 1980's, new features related to the transmission expansion in deregulated environment have attracted researchers' attention. Baldick and Kahn discussed some features of transmission network planning that would possibly cause some controversies in the new environment [14]. The paper classified transmission expansion into two conceptual categories: radial and network connections. Radial expansion involves the connection of two participants where there was no prior interconnection, or the strengthening of a corridor between two participants. Network expansion involves the reinforcement of a power grid. Because the radial approach only looks at the overloaded line and only suggests expanding the overloaded line, it is often not optimal, but it could be implemented easily. Network expansion is generally better than the radial expansion, but it is usually difficult to find out the best expansion scheme. Meanwhile, the authors also discussed the issues of externalities, synergies and economies of scale in the competitive environment. These issues might cause conflicts among transmission service competitors and need to be solved. The authors pointed out that a competitive decentralized economic environment would make the task of transmission planning more difficult. On one hand, optimal network expansion should be achieved, while externalities and economies of scale also should be taken into consideration. On the other hand, transparent information about the expansion should be provided to competing participants to allow them to independently evaluate their options and alternatives. Thus, transmission expansion conflicts could be settled on a voluntary basis or through adjudication. However, all the problems described in [14] are based on the assumption of competitive environment in transmission service. But up to now, nearly all the electrical deregulation models in the world require the transmission system to be regulated and monopolized as before so as to keep the fairness of access to the system. The transmission competition assumed by the authors doesn't exist in present deregulated environment.

Recently, Contreras and Wu [15] have proposed a method called kernel approach to allocate the transmission expansion cost among members of coalition who join together to build the needed new lines. This approach is applicable to a deregulated environment. The kernel is a concept from cooperative game theory that splits a common resource among the players who are willing to join other players to obtain more benefit in terms of the "strength" of the members of the coalition. For the transmission expansion game, this approach will guarantee to reward those members with less cost to the expansion. The main advantages of the proposed method are: higher rewards to stronger players and uniqueness of the solution to the transmission expansion. However, as the authors point out, this proposed approach couldn't solve the allocation of sunk costs. In addition, the expansion cost is allocated among the present coalition members, If a new player uses these new lines several years later, how to pay for the usage of these new lines and how to distribute the benefits again among the old members are not given by the authors.

1.4 Objectives and Scope of the Research

In the traditional vertically integrated monopolic electric industry, it is the utilities' responsibility to expand their transmission systems to meet the demand of load growth and some specific technical requirements such as adequate voltage and reliability. In this case, transmission and generating expansion plans are usually coordinated with each other so that the electric network can operate in an optimal way.

Because no competition exists in regulated systems where system operators have the right to control and dispatch the entire resources including generating and transmission facilities, the electric networks will not be overloaded in a normal operation mode. In order to recover the cost of electric facilities serving in the system, the utilities manage to make full use of them and avoid the stranding of the existing facilities. In other words, if the existing network can meet the operating requirements, the utilities will not expand the system no matter how high the operating cost is. It is because a guaranteed and regulated return makes them less interested to further improve the performance of their systems. Any improvement in the transmission area usually involves the injection of a large amount of fund that would increase the operating cost, which will eventually be allocated among the customers in the form of electricity price that bundles the generation cost and transmission cost together.

Situation is different in the deregulated electric industry. The main goal of deregulation is to encourage competition among power sellers. From the viewpoint of governments, they hope the competition is complete and fair with fewer operating limitations, especially, the limitation coming from the overload of transmission lines. An overload has the potential to hinder electrical power flowing from the area of low price to the area of high price and cause unfair competition. One of the main objectives of power industry deregulation is to permit customers to have more choices to buy electricity from different power producers and forbid system operators to interfere with these trades. In this case, system operators can't schedule the generation any more as they did before. Only the power buyers and sellers can determine the generation schedule according to the competitive market rule. In a competitive market, customer's decision is influenced by the price of electricity irrespective of the operating conditions of the system, and therefore, it is more likely that transmission congestion will occur in some cases. Once the congestion happens, in order to keep the system operating safely, the system operator will have to interfere with it and reschedule the generation to relieve the congestion. Some customers have interruptible loads that can be cut off by system operator to maintain system security. This kind of loads can also be utilized by system operator to relieve congestion if needed. Usually, generation reschedule or interruptible load curtailment will cause system operating cost increase. The increased cost is called congestion fee and usually is undertaken by the customers.

It is an important and challenging task for an ISO to compare the cost of rescheduling generation with the cost of expanding the transmission system. The aim is to relieve the transmission system of congestion. Very little research has been done in this aspect and reported in the literatures. This research is intended to develop a method to assess the potential cost of congestion and suggest whether an expansion of the existing transmission network would be an economic choice. The objectives of this research work were:

- To develop a method to optimize a generation reschedule when congestion happens.
- To develop a method to assess several transmission expansion alternatives in order to find the optimal transmission expansion in case of system congestion.
- To study the influence of interruptible load and short-term power market (PX) on transmission network expansion.

Two approaches to determine a generation reschedule in case of congestion in a fully deregulated power system have been developed and reported in this thesis. One method, called DC optimal approach, is based on neglecting transmission loss and the utilization of DC power flow analysis. This approach is applicable to small systems where transmission loss is negligible. Another method, called AC optimal approach, takes the transmission loss into consideration and utilizes AC load flow analysis. A more satisfactory accuracy can be reached through the use of AC approach and can be used for medium and large systems. Both methods can be used to search for the optimal generation reschedule by examining the relationship between the active power flows through the congested lines and the active output of the generators in a system. Once the generation reschedule is determined, the resulting congestion fee can be assessed. And then, following the proposed procedure described in the thesis, the appropriate transmission lines needed to be built can be selected to solve the congestion problem with an optimal investment.

1.5 Outline of the Thesis

This thesis is organized in six chapters. The main topics of each chapter are described briefly in the following:

Chapter 1 introduces the basics of power system and development of electrical transmission system. It also reviews the recent research progress in the field of transmission system expansion and states the objectives and scope of this thesis.

Chapter 2 discusses some basic issues related to the transmission system expansion problem in the deregulated electric industry. In addition, the way of calculating congestion fee and the method used to search for the optimal expansion planning are also introduced in this chapter.

Chapter 3 presents the development of DC and AC optimal approach for generation rescheduling in details. A test system is utilized as an example to explain the calculation process of determining the optimal generation reschedule by using DC and AC approach respectively. Chapter 3 also describes the modelling of interruptible load and its influence on the congestion problem and transmission system expansion.

Chapter 4 deals with the influence of short-term power market on the transmission expansion planning. This chapter gives out a detailed analysis and discusses on the bidding price function of generators and the components of user's load. The concept of the market-clearing price (MCP) and generation commitments for short-term power market (PX) are also presented in details with a simple example to highlight the calculation process.

Chapter 5 illustrates the application of this research work. It uses an example system to show the entire calculation process for transmission expansion planning from generation prediction to the determination of new transmission lines needed to be added to relieve congestion in the system.

Chapter 6 summarizes the thesis and reports the conclusions.

CHAPTER 2: TRANSMISSION SYSTEM EXPANSION PLANNING UNDER DEREGULATION

2.1 Congestion and Its Management under Deregulation

In recent decade, power industry deregulation has been taking place throughout the world. Deregulation introduces market competition into electrical industry that has been regulated by the government for over one hundred years. Without any doubt, deregulation will inject new activities and market concepts into traditional power industry and make it become more efficient. But deregulation will also bring many new challenges that have never been encountered before and need to be solved.

Under deregulation, a power system is divided into three sections: generation, transmission and consumption. At present, the market competition is mainly focused on generation section. According to the principle of deregulation, each generator must bid its price to sell electrical power in the short-term power market or make a deal to sell power to customers privately. Thus, customers will have more options to buy cheaper power to meet their load demands from different power sellers. Although generation market is deregulated to encourage competition between power sellers, transmission section is still regulated by government to ensure each power seller and buyer has equitable access to the transmission system. A transmission grid might be accessed by many different utilities. In order to ensure the fairness of power market competition, an impartial organization called Independent System Operator (ISO) is usually established to monitor and operate the interconnected power system that extends over a wide region.

An important aspect of a typical ISO is that it runs, but does not own the power system. An ISO has four major responsibilities with respect to the power grid: system security, delivery of power, recovery of transmission cost, and fair access to transmission system.

In deregulated power industry, electricity trades, which are also called electrical transactions, are often taking place privately between power producers and power purchasers, or through power exchange market. An ISO has no right to schedule generation as a traditional system operator did before. This functional change inevitably makes it difficult to manage the resources and control the system operation by an ISO. Power sellers and buyers are, however, blind to system operation conditions. In this case, they can't guarantee that all their trades can meet the system operation requirements at any time. Because, these trades are completely the results of market competition, with which the ISO has no right to interfere. Sometimes, these trades will cause some electrical transmission lines to be overloaded. This phenomenon is called transmission system congestion which can be explained by using the following example.

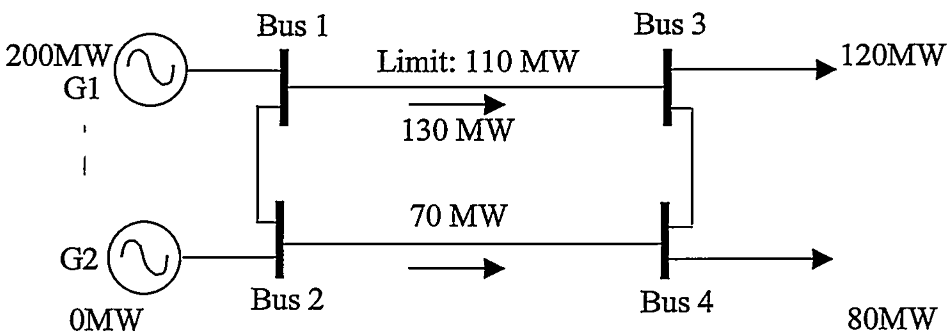


Fig. 2.1 Transmission System Congestion

Consider the system shown in Fig. 2.1. It consists of four buses. There is a generator with capacity of 200 MW at Bus 1 and a generator with capacity of 120 MW at Bus 2. Assume that G1 has lower generation cost than G2 and all transmission lines have same resistance and reactance. Also assume that Line 13's operating limit is 110

MW and the remaining three lines have enough capacity to carry any load. There are two loads in this system; Load 1 is 120 MW and Load 2 is 80 MW. Under a deregulated system environment, competition will make Generator 1 win and take over all 200 MW of load due to its lower generation cost. If transmission loss and reactive power is ignored, power flows in Line 13 and Line 24 could be obtained by employing DC power flow analysis technique [18]. The line flows through Lines 13 and 24 are 130 MW and 70 MW respectively. Obviously, Line 13 is overloaded by 20 MW, or say, Line 13 is congested due to electrical market competition.

Transmission system congestion is an unavoidable consequence brought by power industry deregulation. The reason is very simple and understandable. From the standpoint of customers, cheaper power has greater attraction to them. Therefore, every customer hopes to buy electricity from those power plants with lower electricity prices. In other words, it is the price that determines the electrical transactions, generation schedule and load configuration, not the system operating conditions. Congestion left unsolved will threaten the security and safety of system operation such as damaging transmission line due to overheating and causing some stability problems. An ISO has to take some necessary measures to prevent transmission congestion. The measures taken by an ISO usually include decreasing the outputs of some generators that are supplying cheaper power while raising the outputs of other generators with high electricity prices, and if necessary, curtailing some interruptible loads to relieve the congested transmission lines. For example, to relieve the congestion in system shown in Fig. 2.1, its ISO will order G1 to decrease its output and at the same time require G2 to increase its output. Thus, power flow in Line 13 will be reduced. But rescheduling generation and load will cause system operating cost to go up. Because, not only the generators ordered to increase outputs should be paid, but also those generators (or customers) decreasing their outputs (or loads) must be compensated too. The extra cost will be allocated among customers who cause the congestion in terms of their usage capacity of overloaded transmission lines.

Formula (2.1) may be used to calculate the congestion cost for each user [16]:

$$CF_i = \frac{P_i}{\sum_i P_i} X\, C_j; \qquad (2.1)$$

C_j congestion cost allocated to a congested transmission line j.

P_i power flow in transmission line j due to transaction i.

CF_i congestion cost allocated to transaction i.

P_i will be negative if its direction is opposite to the actual power flow in Line j. This situation is called counter-flow. Some papers suggest that those transactions causing counter-flows should be compensated because they make contributions to congestion relief. But others think that counter-flow is a concept relative to other flows. Because the counter-flow probably would also cause congestion if the positive flow didn't exist. Therefore, they suggest that transactions introducing the counter flows shouldn't be paid; meanwhile, they also needn't pay for congestion.

The total congestion cost for each user is [16]:

$$TCF = \sum_{i=1}^{k} CF_i \qquad (2.2)$$

k the number of congested lines

2.2 Transmission System Expansion Cost Recovery

If the congestion fee is too high and an ISO has no other resources to use to decrease it, installing new transmission line will become the only way to reduce the cost.

In deregulated power industry, the following facts should be taken into account when ISO make a decision as to whether or not to expand the existing transmission system:

1) In order to obtain the power supply at a stable and favorable price, most customers would like to make a long-term deal with power sellers for contracted energy quantities. This may result in long-term contracts (for example: 10 years or 15 years) for electricity trades. Cancellation of these trades to re-contract with other power suppliers might be impossible and unacceptable in practice.

2) Long-term curtailment of firm load is also impossible and unacceptable from the viewpoint of customers. It prevents the customers from developing their production, and reduces the customers' profit.

3) Installation of new electrical transmission equipment of adequate sizes will eliminate the congestion and it will bring down the cost that customers need to pay if the total cost of building and operating these new transmission lines is less than the congestion fee.

The planning of transmission system expansion usually involves assessment of investment cost recovery and profit return. The costs associated with an expansion will be distributed among the customers who would use the new lines. In order to protect the users' interest and avoid unnecessary expansion, an ISO should be endowed with the right to determine when and where the transmission system expansion should take place and what amount of new transmission capacity is needed. Several methods are used to distribute the system expansion cost among the users, such as stamp postage method employed in California system [4]. This method ignores transmission distance in allocating transmission cost among users and set a transmission price on use of the grid that depends only on the amount of power moved. In other words, no matter how long the powers required by transactions go through and how many lines are needed to move these powers, those transactions will pay the same amount of money if they require to transfer the same amount of power. This method is simple and easy to implement, but it is unfair for short-distance transactions, which need fewer lines than long-distance transactions, and especially, unfair for those transactions that don't need to use the new lines. It seems more equitable to allocate the expansion cost among the users of the new transmission lines in proportion to their capacity usage. Because, only these customers would enjoy the benefits brought by the expansion of transmission system, such as higher reliability. The following formula may be used to calculate the transmission expansion cost allocated to each user [17]:

$$UC_i = \frac{|P_i|}{\sum_i |P_i|} X C; \qquad (2.3)$$

C cost of a new transmission line.

P_i active power flowing through the new transmission line caused by transaction i.

UC_i cost allocated to transaction i.

The reason that the absolute value of P_i is used in (2.3) is that transaction i need to pay for transmission fee for using the new line even if it causes counter-flow. It is better to charge buyers rather than charging buyers and suppliers both. If the suppliers are charged, they will insert this cost into generation cost and manage to transfer it to buyers through higher electricity price. It has the same result as charging buyers only. In addition, when power sellers negotiate with buyers, they usually don't know how much money they should pay for using the new transmission line. In order to avoid this cost, they have to make an estimate and transfer this estimated cost to buyers. Because this estimation is not accurate, sometimes, overestimated, the buyers will assume unreasonable cost.

The cost of a transmission line can be decomposed into three parts: investment cost, benefit return cost and maintenance cost. Investment cost, which is equal to equipment purchase cost plus installation cost, should be recovered within the years that the shortest long-term bilateral transaction spans over, because it is not sure if this bilateral transaction will continue to exist after its expiry, or if other bilateral transactions will take place. If the shortest bilateral transaction didn't continue after its expiry, maybe the congestion would not exist. In other words, the future is uncertain. Any equipment has its own usage life, for example, 20 years. If the investment cost is recovered within the usage life of the transmission line, obviously, the longest bilateral transaction will have to undertake most of the investment cost. It is unfair and unacceptable, because the longest transaction is not the only one causing congestion. As for the benefit return cost, which can be defined as a regulated and guaranteed level of profit for the line owners (for example, total 50% interest return is guaranteed to be paid to the line owners as a profit through over the line usage life), it is better to distribute it along the whole usage life of the line. The reason is that this line will serve in the system until it reaches its usage life. During this period, maybe other bilateral transactions will join to use this line; therefore, it is better for them to share the service fee with the existing transactions. The longer bilateral transactions enjoy longer transmission service brought by this line and it is, therefore, reasonable for them to pay more towards the service fee. Maintenance cost is almost the same every year if the inflation is not taken into consideration. It should be paid by all customers using this line year by year. Therefore, the yearly cost of a new line before the expiry of the shortest transaction can be expressed as:

$$\frac{\text{Total Investment Cost}}{\text{Years of Shortest Long-Term Transaction}} + \frac{\text{Total Benefit Return Cost}}{\text{Usage Life of the New Line}} + \text{Yearly Maintenance Cost} \tag{2.4}$$

The yearly cost of the new line after the expiry of the shortest transaction is:

$$\frac{\text{Total Benefit Return Cost}}{\text{Usage Life of the New Line}} + \text{Yearly Maintenance Cost} \tag{2.5}$$

As soon as the yearly cost of a new line is determined, we can obtain the hourly cost of this line through dividing the yearly cost by the total number of hours of one year minus the maintenance hours. Then, we charge the users hour by hour in terms of their capacity usage of this new line. Thus, those consumers, who just use the new transmission line for only a few hours, will also pay for the expansion cost. It, obviously, will reduce part of financial burden from those customers having long-term transactions and seems to be more reasonable. The hourly cost of a new line can be expressed as:

$$C = \frac{YC}{8760 - MT} \tag{2.6}$$

YC yearly cost of new line.

MT yearly maintenance hours of new line.

In order to increase the usage efficiency of a transmission line, the policy of distributing the transmission cost should encourage customer to consume power at the time of light load. It means charging more at peak load and less at light load. Thus, some customers, especially, some industry customers will likely to arrange part of their loads at off-peak time to decrease their production costs. The benefit of this policy is that the transmission capacity is used fully and some unnecessary constructions can be avoided. A day can be divided into several time segments. Each time segment will have different charge for power transmission.

Transmission fee in each time segment can be calculated as:

$$C_i = a_i * C; \qquad (2.7)$$

C average hourly cost obtained from (2.6);

a_i charge coefficient for each time segment.

$(\sum_{i=1}^{n} a_i) / n = 1$, n is the number of time segments per day.

C_i electricity charge per hour for each time segment.

2.3 Power Market Model

Only two types of transactions are considered in this thesis: bilateral transactions and multilateral transactions.

In a bilateral transaction, a power seller and buyer enters into transactions where the volume and price are decided by the two parties without the intervention of an ISO. Bilateral transactions are brought to the ISO with requests that the necessary transmission be provided. If there is no congestion in the system, the ISO simply dispatches all requested transactions and makes a fair charge for the service. So long as the ISO can accommodate all the requested transactions without technical difficulties, bilateral contracts and the ISO operation are decoupled from each other except for the need for the ISO to make up transmission losses and charge for ancillary services.

A bilateral transaction can be expressed as a simplified mathematical model. Let P_{ij} be the power transaction requested from seller bus i to buyer bus j. For convenience, seller buses are numbered from *2* to m and buyers are assumed to be located at bus $m+1$ to n. Bus *1* is reserved for the ISO to purchase power to make up transmission losses. Since buyers and sellers may enter into a multitude of transactions where each buy and sell from many others, the total power at bus i and j are given by:

$$
\begin{aligned}
P_i &= \sum_j P_{ij} \qquad & i = 2,\dots,\mathrm{m} \\
P_j &= \sum_i P_{ij} \qquad & j = \mathrm{m}+1,\dots,\mathrm{n}
\end{aligned}
\tag{2.8}
$$

A bilateral transaction is called a group transaction where a certain number of generating plants and power purchasers combine together to form a group. A utility, which may own a number of power stations at diverse locations and serves customers also at various locations, may become part of a group transaction. In the event of a system congestion, the utility may be willing to reschedule generation among its own generators provided it could keep all its customers supplied. This kind of generation reschedule doesn't bring any extra cost to system operation. In other words, it helps the ISO eliminate the congestion problem, but doesn't require any compensation from ISO for this generation reschedule. This type of generation reschedule was not considered in this thesis.

Conceptually, a multilateral transaction differs from the bilateral transaction in both its contractual mechanisms and its operational implications to the ISO. In a multilateral transaction, the primary parties, the buyers and sellers, do not make contracts directly between each other. A third party, such as a broker in power exchange market, will put together deals between several power sellers and buyers. A typical example is Alberta's Power Market, where sellers bid their prices for power supply and buyers bid their prices for power demand. Successful participants make a deal at the same price --- market-clearing price (MCP). The broker submits its total set of buyer and seller schedules to the ISO to request power delivery. If the requested power deliveries do not cause any congestion in the system, the ISO simply dispatches them. Actually, this kind of multilateral transaction can be viewed as a special bilateral contract in the eyes of an ISO. Therefore, in this thesis, this type of transaction is also treated as a bilateral transaction.

2.4 Congestion Cost Calculation

To simplify the calculation of a congestion cost, following assumptions have been made.

1) System generating capacity is enough to support all load demands. Therefore, any generation expansion is not considered. Actually, generation expansion has great impact on the transmission system expansion. Generation expansion will make the transmission system expansion more complicated due to its uncertainty.
2) Only feasible transactions are considered. It means that, although execution of these transactions will cause congestion, the existing system has enough power sources or interruptible loads to relieve the congestion without curtailing customers' firm loads. If congestion elimination, however, relies on customers curtailing their firm loads for a long time, construction of a new transmission line is needed without any hesitation.

3) The loads used to calculate the congestion cost are established on the basis of bilateral and multilateral transactions. Consider the transactions that exist at the time when one decides whether a new line is needed or not. In other words, do not consider the future transactions because of their uncertainty and unpredictability.

4) There is only one slack bus in the system. The ISO uses this bus to buy power to make up for the transmission loss.

5) Only normal operation of transmission system is considered in this research. Operation under contingency is not taken into account, because, it is only a temporary operation schedule with only short-term impact on the system operation.

6) Construction of backup transmission lines to strengthen the system operation security and reliability is not considered.

7) All generators taking part in competition in power market must submit their generation redispatch prices to the ISO. Thus, the ISO can use this information to reschedule optimal generation commitments when a system congestion happens.

The cost of relieving congestion, known as congestion fee can be calculated in the following manner.

First, the ISO runs a power analysis program from 00:00 a.m. in terms of the transactions submitted by the participants and checks if there is a congestion in the system. If no congestion occurs at this hour, the ISO simply dispatches the submitted transactions at this hour without making any change, and no congestion fee is charged by the ISO.

If there is a congestion at this hour, the ISO will have to reschedule the generation to relieve the congestion. In this situation, some actions are necessary to solve this insecure problem. These actions include:

1) Use of an appropriate OPF program to determine the optimal generation reschedule without violating any security constraint.
2) Appropriate changes in generation schedules, ordering some generators to increase, and the others to reduce output or curtail interruptible load, until the congestion is eliminated.
3) Compensation for the parties who were asked to generate more, effectively paying them for their additional power production. Generators (customers) who are required to cut back their outputs (interruptible loads) are granted with "lost opportunity payments".

The ISO can collect congestion fee from the users who caused the congestion to compensate affected generators (customers).

Usually, the compensation amounts given to the affected generators are determined in a free market manner. Generators connected to the system are required to submit "increment and decrement price for power" to the ISO. For example, a generator is willing to increase its output at 6 cents per kWhr, and is willing to reduce its output by paying out to the ISO at 2 cents per kWhr. If this generator has a transaction with a customer at 4 cents per kWhr, it can get 6 cents per kWhr for extra increased output and it still get 2 cents per kWhr (4 cents – 2 cents = 2 cents) for the reduced output as a compensation for lost opportunity payment. The ISO uses these submitted prices to obtain the optimal generation reschedule through an appropriate OPF program. Once the optimal generation reschedule at this hour is found out, the congestion fee at this hour can be evaluated by using the following expression:

$$CF_i = \sum_{i=1}^{k} C_j * \Delta P_j \qquad (2.9)$$

Where,

$$\sum_{i=1}^{k} \Delta P_i = 0;$$

CF_i congestion fee at hour i;

C_j redispatch price submitted by generator j;

ΔP_j active output change of generator j required by ISO;

k number of generators;

The analysis is repeated for all 24 hours. The total congestion fee for a whole day is found by adding all congestion fees at different hours.

$$CF = \sum_{i=1}^{k} CF_i \; ; \qquad (2.10)$$

CF total congestion fee for a whole day;

k = 1, 2, 3, ……, 24;

2.5 Optimizing the Transmission Expansion Plan

Transmission system expansion planning under deregulated environment is different from traditional expansion planning. In the past, power system is integrated vertically. A system operator had right to dispatch all available resources in the system to meet the load demand no matter how high the operating cost was. Customers had to pay for it. Under the old system, if the generation and transmission configuration could keep the system operating normal and secure, the power company would have no interest to expand the system no matter how high the operating cost is, because the high operating cost can be recovered from electricity charges. Only when system condition cannot meet the load growth, system expansion was taken into consideration.

The following mathematical model is used to describe a traditional transmission system expansion-planning problem [13]:

$$\text{Min } v = \sum_{ij} C_{ij} * n_{ij} + \sum_{i} \alpha_i * r_i \tag{2.11}$$

Subject to:

$$B(b+\gamma^0)\theta + g + r = d;$$
$$f_{ij} - (\gamma_{ij}^0 + b_{ij})(\theta_i - \theta_j) = 0;$$
$$|f_{ij}| - b_{ij}\phi_{ij}^{\max} \le \gamma_{ij}^0 \phi_{ij}^{\max}; \quad \phi_{ij}^{\max} = f_{ij}^{\max}/\gamma_{ij}^0;$$
$$0 \le g \le g^{\max}; \ 0 \le r \le d;$$
$$b_{ij} = n_{ij}\gamma_{ij}; \quad 0 \le n_{ij} \le n_{ij}^{\max};$$

Where,

C_{ij} investment cost of building a new circuit in branch i-j;

n_{ij} number of circuits added in branch i-j;

α penalty parameter associated with loss of load caused by lack of transmission capacity;

r array of load curtailments;

$B(.)$ susceptance matrix;

G array of bus active powers;

d array of predicted bus loads;

f_{ij} active power flow through branch i -j;

γ_{ij}^0 initial susceptance in branch i -j;

b_{ij} total new circuit susceptance added to the branch i -j;

γ_{ij} circuit susceptance;

θ_i, θ_j voltage angle at bus i and j;

$f_{ij}^{\max}$ flow limit in branch i-j;

$g^{\max}$ maximum bus generation capacity array;

$n_{ij}^{\max}$ maximum number of new circuits in branch $i-j$;

The objective function (2.11) represents the investment cost of building new transmission facilities plus a higher penalty for load curtailments. Typically, the parameter α is determined from statistical studies that identify the impact caused to customers by shortages of electricity. α is the maximum price that the consumers are willing to pay for a continuous supplying of electricity.

Under deregulation, traditional expansion method is not suitable for the new market environment. Whether a new transmission facility should be built or not isn't determined by the power companies or transmission companies in a deregulated environment. Sometimes, the system has enough generating capacity to meet the demand of customers, but driven by the profit, each customer wants to purchase power from those generation companies providing cheap electricity. This trade behavior will cause some generators with high generation cost become idle, and also cause some transmission lines not to be used fully, and at the same time, some other transfer facilities may be overloaded, or say, congested.

In order to relieve congestion and keep a system operating safely, an ISO has to reschedule some generators' output. This interference generally increases the operating cost. When the increased cost is higher than the investment cost of building new transmission facilities, the ISO should consider adding new transfer equipment to the system to bring down customers' electricity transmission cost. Therefore, under deregulation, addition of new electrical facilities is completely dependent on the market behaviour. This is different from the traditional system expansion planning.

The simplest way to relieve congestion is to build new transmission facilities in parallel with the overloaded ones. But sometimes, this addition scheme is not the best solution. An optimum solution would solve the overload problem and at the same time would require the least investment cost.

As an illustration, the system shown in Fig. 2.2 is considered. It consists of four nodes. There is a generator with capacity of 250 MW at Node 1, a generator with capacity of 100MW at Node 4, 100 MW of load at Nodes 2 and 4 and 50 MW of load at Node 3. Assume that the transmission line losses and reactive power flows are ignored and that the cost of building additional transmission line directly between any pair of Nodes 2, 3 and 4 is approximately the same. This would be the case if the nodes are equidistant, as illustrated, and if the terrain and environmental considerations are the same for each of the possible corridors. Also assume that the transmission limits of line 2-3 and line 3-4 are 100 MW and 50 MW and G1 has lower generation cost than G2.

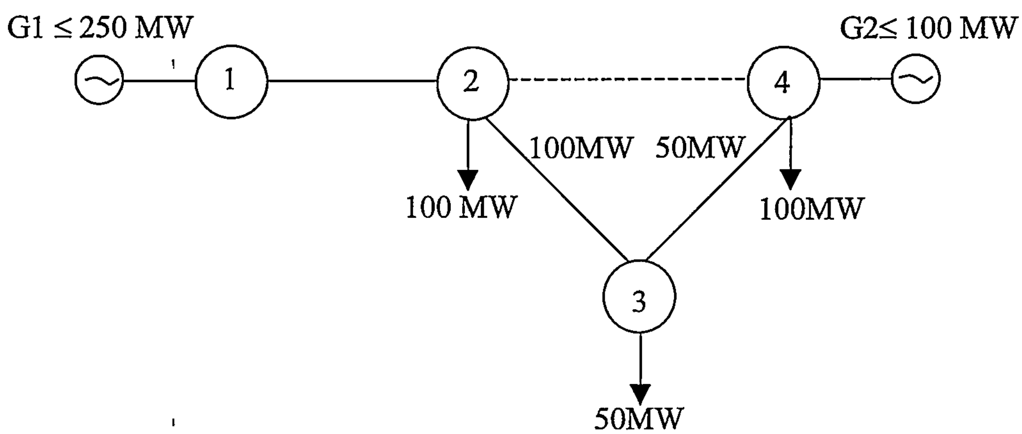


Fig 2.2 Transmission System Expansion under Deregulation

Under a regulated power system environment, the economic operation mode (operation constraints are considered) is: G1 generating 200 MW and G2 contributing 50 MW. Thus, the power flows through Lines 23 and 34 are 100 MW and 50 MW. No overload exists in the system. But under a deregulated system environment, competition will make Generator 1 win and take over all 250 MW of load due to its lower generation cost. Generator 2 will stop supplying. In this case, power flows in Lines 23 and 34 increase up to 150 MW and 100 MW respectively. These two lines are overloaded by 50 MW and a generation reschedule is needed to alleviate the overload condition. If the congestion fee is too high and the ISO in charge of the system considers an expansion of this system, the simple-minded transmission expansion planning will be to build two transmission lines in parallel with Line 23 and 34 respectively. This would be a feasible plan, however, a better plan is to build a new line between Nodes 2 and 4 as represented by the dotted line in Fig. 2.2. In this case, the power flows in Lines 23 and 34 will become 67 MW and 17 MW respectively. Congestion problem is solved with only half of the investment cost of the first suggested plan. But, the question is how to find out the optimal expansion plan if congestion cannot be alleviated simply by rescheduling system generation.

2.5.1 Optimal Expansion Planning with DC Power Flow

A search for optimal expansion planning can be done using DC power flow analysis [18]. This method is based on DC power flow equations. These equations, usually *N-1* in number, where *N* is the number of buses, model only real power flows and ignore reactive flows. In many cases, all line resistances are neglected, and most of the line flows are assumed to be considerably smaller than the steady-state stability limit or thermal limit of the line. This assumption is acceptable, because a transmission network is generally robust enough to meet the load demand in most cases. Congestion only happens in a few transmission facilities. The result of all these assumptions is that a linear model of the network can be obtained to facilitate the analysis. To develop DC power flow equations, the standard AC power flow equations must be employed:

$$P_i = \text{Real}\,(V_i^* \sum_j Y_{ij}V_j) \qquad (2.12)$$

$$Q_i = -\,\text{Im}(V_i^* \sum_j Y_{ij}V_j) \qquad (2.13)$$

$$i = 1, 2, \ldots, N$$

Assuming the voltage magnitudes remain unchanged and are equal to their base case values, the N net injected reactive power equations can be ignored, leaving only the N real power equations. Since one of the buses must be specified as reference bus, one of the equations in (2.12) can be deleted. Deleting the equation corresponding to the slack bus, which is defined as Bus 1, the active power injected into Bus i will be:

$$\begin{aligned} P_i &= \text{Real}\,(V_i^* \sum_j Y_{ij}V_j) \qquad i = 2, 3, \ldots, N \\ &= \text{Real}\,(|V_i|e^{-j\delta_i} \sum_j |Y_{ij}|e^{j\theta_{ij}}|V_j|e^{j\delta_j}) \\ &= \text{Real}\,(\sum_j |V_i||V_j||Y_{ij}|e^{j(\theta_{ij}+\delta_j-\delta_i)}) \end{aligned}$$

With the line resistance neglected, the admittance is:

$$Y_{ij} = G_{ij} + jB_{ij} = jB_{ij}$$

and

$$P_i = \sum_j |V_i||V_j|B_{ij}\sin(\delta_i - \delta_j) \qquad i = 2, 3, \ldots, N$$

Generally, $\delta_i - \delta_j$ is sufficiently small that the sine of the angle difference can be replaced by the angle difference. Thus:

$$P_i = \sum_j |V_i||V_j|B_{ij}(\delta_i - \delta_j)$$

$$= \sum_{j=1}^{i-1} |V_i||V_j|B_{ij}(\delta_i - \delta_j) + \sum_{j=i+1}^{N} |V_i||V_j|B_{ij}(\delta_i - \delta_j)$$

$$= - \sum_{j=1}^{i-1} |V_i||V_j|B_{ij}\delta_j + \delta_i \sum_{j \neq i} |V_i||V_j|B_{ij} - \sum_{j=i+1}^{N} |V_i||V_j|B_{ij}\delta_j$$

Let's assume $|V_i| = |V_j| = 1.0$, we have:

$$P_i = - \sum_{j=1}^{i-1} B_{ij}\delta_j + \delta_i \sum_{j \neq i} B_{ij} - \sum_{j=i+1}^{N} B_{ij}\delta_j \qquad i = 2, 3, \ldots, N$$

Or in matrix form:

$$P = B\delta$$

where, P is the vector of bus power injection, B is the bus admittance matrix and δ is the vector of bus angles.

Clearly, if for a fixed set of power injections P, a line or lines are added to the system, both B matrix and δ vector will change from their base-case values B^0, δ^0 by an amount ΔB and $\Delta\delta$ such that:

$$P^0 = (B^0 + \Delta B)(\delta^0 + \Delta\delta)$$

$$= B^0\delta^0 + \Delta B\delta^0 + B^0\Delta\delta + \Delta B\Delta\delta$$

If the higher order term $\Delta B\Delta\delta$ is dropped, then,

$$P^0 \approx B^0\delta^0 + \Delta B\delta^0 + B^0\Delta\delta$$

Because $P^0 = B^0\delta^0$, the following expression can be obtained:

$$\Delta\delta = - (B^0)^{-1} \Delta B\delta^0 \qquad (2.14)$$

Equation (2.14) provides the changes in the bus voltage angles due to network changes. Since the change in the power flow in Line ij is:

$$\Delta P_{ij} = - B_{ij}(\Delta\delta_i - \Delta\delta_j) \qquad (2.15)$$

where, $\Delta\delta_i$ and $\Delta\delta_j$, are the ith and jth components of $\Delta\delta$. The effects of line additions on line flows are readily available.

It is important to emphasize that $(B^0)^{-1}$ need only be computed once at the beginning. As long as the base case remains the same, $(B^0)^{-1}$ is valid. If a single transmission line is added, for example, Line *mn*, then the only elements in B^0 that change are B_{mm}, B_{mn}, and B_{nn}, where

$$\begin{aligned}
\Delta B_{mm} &= B_{mm}; \\
\Delta B_{nn} &= B_{nn}; \qquad\qquad (2.16) \\
\Delta B_{mn} &= -B_{mn}; \\
\Delta B_{nm} &= -B_{mn};
\end{aligned}$$

Substituting the information in Equation (2.16) into Equation (2.14), we have:

$$\Delta\delta = -(B^0)^{-1}\,\Delta B\,\delta^0$$

$$= -(B^0)^{-1} \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot \\ 0 & \cdots & B_{mn} & \cdots & -B_{mn} & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot \\ 0 & \cdots & -B_{mn} & \cdots & B_{mn} & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdots & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \delta^0$$

$$= -(B^0)^{-1}\,(\delta_m^{\,0}\Delta B_m + \delta_n^{\,0}\Delta B_n)$$

Where ΔB_m and ΔB_n represent the mth and nth columns of the ΔB matrix. The changes in line flows due to the addition of line *mn* are obtained by substituting the appropriate elements of $\Delta\delta$ into Equation (2.15). In this way, we easily obtain the effects of all single transmission line addition on system power flows, and we can use this information to determine where a transmission line should be built to relieve the congestion and what is the best investment scheme.

The effects of adding double or more transmission lines to system is straightforward since Equation (2.14) is a linear function of the network change ΔB. If we want the change in line flows that will occur with the addition of both Lines ij and mn, we can easily obtain the change in line flows through simple linear superposition:

$$\Delta\delta = \Delta\delta^{ij} + \Delta\delta^{mn}; \qquad (2.17)$$

Where $\Delta\delta$ is the vector of the change of bus voltage angles after adding transmission Lines ij and mn. $\Delta\delta^{ij}$ and $\Delta\delta^{mn}$ are vectors of the change of bus voltage angles after adding transmission Lines ij and mn respectively. The superposition shown in Equation (2.17) can be derived in the following manner.

$$\Delta\delta = -(B^0)^{-1}\,\Delta B\,\delta^0$$

$$= -(B^0)^{-1}\begin{bmatrix}
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & B_{ij} & \cdots & -B_{ij} & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & -B_{ij} & \cdots & B_{ij} & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & B_{mn} & \cdots & -B_{mn} & \cdot & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & B_{mn} & \cdots & -B_{mn} & \cdot & 0 \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdot\cdot & 0
\end{bmatrix}\delta^0$$

$$= -(B^0)^{-1}\,(\delta_i^0\Delta B_i + \delta_j^0\Delta B_j + \delta_m^0\Delta B_m + \delta_n^0\Delta B_j)$$

$$= -(B^0)^{-1}\,(\delta_i^0\Delta B_i + \delta_j^0\Delta B_j) - (B^0)^{-1}\,(\delta_m^0\Delta B_m + \delta_n^0\Delta B_j)$$

$$= \Delta\delta^{ij} + \Delta\delta^{mn}$$

For any existing transmission Line pq, the power flow change ΔP_{pq} with the addition of transmission Lines ij and mn is:

$$\begin{aligned}\Delta P_{pq} &= -B_{pq}(\Delta\delta_p - \Delta\delta_q) \\ &= -B_{pq}(\Delta\delta_p^{\ ij} + \Delta\delta_p^{\ mn} - \Delta\delta_q^{\ ij} - \Delta\delta_q^{\ mn}) \\ &= \Delta P_{pq}^{\ \ ij} + \Delta P_{pq}^{\ \ mn}\end{aligned}$$

where the $\Delta P_{pq}^{\ \ ij}$ and $\Delta P_{pq}^{\ \ mn}$ are the power flow changes in Line pq with the addition of transmission Line ij or mn respectively.

DC power flow analysis is the simplest, but inaccurate method to determine line flows with a change of network structure. It is not suitable for power flow analysis in actual operation, but feasible and pragmatic for system expansion planning, because the latter has no strict requirement on the accuracy of power flow data. The only difference between a DC power analysis and an AC power analysis is that the DC approach neglects transmission loss. Although total transmission loss in a system can reach up to a high value in some cases, for example, 15%, the transmission loss in each line is generally much lower than this value. As known, under normal operation, the voltage difference between two ends of a line is generally below 10% of rated voltage. A formula widely used to calculate transmission loss in a transmission line is described in [3] as:

$$Line\ Loss = \frac{P^2 + Q^2}{U_e^2} R \tag{2.18}$$

where, P is the active power and Q is the reactive power flowing through the line, R is the line resistance and U_e is the rated voltage of line. Generally, according to practical operation requirements, the power factor of a load flow in a transmission line must be above 0.9 so as to reduce transmission loss. Thus, we have:

$$Q \le \frac{\sqrt{1^2 - 0.9^2}}{0.9} P = 0.4843P \qquad (2.19)$$

Substituting Equation (2.19) into Equation (2.18), line loss can be obtained:

$$Line\ Loss \le 1.2345R\ \frac{P^2}{U_e^2} \qquad (2.20)$$

A formula used to calculate the voltage loss across a line is described in [19] as:

$$Voltage\ Loss \approx \frac{U_e}{PR + QX} \qquad (2.21)$$

where X is the line reactance. If X can be related to R as: $X = \mathrm{k}R$, then:

$$Voltage\ Loss = (1 + 0.4843\mathrm{k})R\,\frac{P}{U_e} \le 10\%U_e$$

$$R\,\frac{P}{U_e^2} \le \frac{0.1}{1 + 0.4843\mathrm{k}} \qquad (2.22)$$

Substituting (2.22) into (2.20), then,

$$Line\ Loss \le 1.2345\,\frac{0.1}{1 + 0.4843\mathrm{k}}\,P$$

$$\frac{Line\ Loss}{P} \le 0.12345/(1+0.4843\mathrm{k})$$

Generally, in a transmission system, line X is much greater than R, k $\approx$ 4 ~ 6, therefore, the loss percentage in a transmission line is less than 3.16% ~ 4.2%. It shows that, for a line, if its transmission loss is ignored, the maximum power flow error is only 4.2% of the actual power flow in this line. It indicates that the usage of DC power flow analysis is acceptable in system expansion plan calculations.

Before a search for optimal expansion plan is started, it is needed to acquire some investment cost information for each possible new connection. Generally, the investment cost for transmission line is determined by its length, voltage level, capacity and location. In our research, it is assumed that all these data are known. To simplify the problem, only a single transmission network is considered, for example, a provincial transmission network. Different networks may be connected to each other through some interconnection lines. A network may have contracts with other network for power exchanges, which should be limited within specified range, thus, we can consider these power exchanges as generator's output or customer's load. At the same time, we can treat the transmission networks with deferent voltage levels separately.

As we stated before, the simplest way to relieve congestion is to build new transmission lines in parallel with the overloaded lines. But sometimes, this addition scheme is not the best. In the process of searching for optimal expansion planning, we only consider those possible connections with investment cost lower than the congestion fee. The search for an optimal expansion involves the following steps:

1) Form the B^0 matrix on the basis of the existing network structure.
2) Use a power flow analysis program to obtain the bus voltage angles and power flow in each transmission line without considering the transmission line constraints. Use these angles as δ^0.
3) If congestion occurs, use an OPF program to reschedule the generation and evaluate the congestion fee for a single day.
4) Mark those possible connections with investment cost lower than the congestion fee for a single day.
5) Start from the lowest investment cost. Keeping the original generation schedule unchanged, add a possible Line pq to the system, and then, apply the DC power flow analysis method to obtain the change of power flow in the

overloaded Line *ij*, $\Delta P_{ij(pq)}$. Obtain the average investment cost for per MW transfer capacity increase in Line *ij*:

$$\Delta C_{ij(pq)} = \frac{C_{pq}}{\Delta P_{ij(pq)}} \tag{2.23}$$

where, ΔC_{ij} is the average investment cost for per MW transfer capacity increase in Line *ij*; C_{pq} is the investment cost of newly added Line *pq*.

6) Select the next possible line and repeat step 4 until all possible lines are considered.
7) Addition of new Line *pq* that has the minimum $\Delta C_{ij(pq)}$ and can relieve the congestion in Line *ij* is the best expansion plan for Line *ij* transfer capacity increase.
8) Repeat steps 4 to 6 until the best expansion plans for all overloaded lines are found.
9) Sum all investment costs for all newly added lines; if the total investment cost is less than the total congestion fee, then, this expansion plan is the best.

If no appropriate scheme is found, it indicates that paying for congestion fee is more economical than paying for new line investment cost. Therefore, the addition of new transmission line is not necessary, because the customers would rather pay for congestion fee than building a new transmission line to relieve the overload.

Generally, a transmission system is built in a robust manner. Therefore, if congestion occurs, it only happens in a few lines, and usually, the overloads are such that addition of a few lines is enough to solve the congestion problem. If the congested lines are far away from each other and are little related to each other in power flow distribution, we could treat them as isolated congestion problems. Isolated congestion problems are straightforward and easy to solve.

2.6 Optimal Power Flow Programming

When congestion occurs, an ISO has to re-schedule the generation to keep the system security and stability. This re-scheduling should ensure that the increased cost is the lowest possible increase, in other words, this change should be based on the optimal cost. Meanwhile, under deregulation, an ISO is required not to interfere with the original generation schedule too much, otherwise, the rules of competition and free market will lose meaning. Therefore, the traditional optimal power flow programs are not applicable to the new situation due to the following reasons:

1) A traditional optimal power flow program is not based on the market competition. It makes use of the cost function of each generator, which is generally represented by a quadratic function with the active output as the variable, to form the objective function. It is only suitable to an integrated system where most of the generation resources belong to one power company and controlled by system operators. In this case, the system operators determine which generator contributes what amount of active power according to the optimal principle. But under deregulation, power sellers and buyers determine the basic generation schedule. A system operator can only re-schedule the generation on the basis of price bids.
2) In a free market, a generator's cost function is a private information. A generation company doesn't want to share this important information with

others. Therefore, the ISO in charge of the system operation will not be able to obtain this information from the power sellers.

Due to the above reasons, the traditional OFP methods have to be given up. Two new methods are proposed and introduced in this thesis to obtain optimal generation to eliminate congestion. These two methods, called DC optimal approach and AC optimal approach, are described in details in Chapter 3.

CHAPTER 3: OPTIMAL POWER FLOW METHODS ON THE BASIS OF BIDDING PRICES

3.1 DC Optimal Power Flow Programming Method

When congestion occurs, an ISO has to reschedule the generation under its jurisdiction to keep system operation secure and safe. This rescheduling should ensure that the increased cost is the lowest. Meanwhile, under deregulation, an ISO is required not to interfere with the original generation schedule too much in order to comply with the rules of competition and free market. The concepts related to traditional optimal power flow cannot be applied in a deregulated system.

A proposed method called DC method is introduced here that can be utilized to obtain an optimal generation schedule with demand of eliminating the congestions. This method employs DC power flow analyses to determine a re-scheduled generation commitment.

3.1.1 Mathematical Development of DC Method

If transmission loss is neglected, the mathematical model for optimizing the rescheduled generation can be described as below:

$$Min \sum_{i=1}^{k} C_i * \Delta P_i \qquad (3.1)$$

Subject to:

$$\sum_{i=1}^{k} \Delta P_i = 0;$$

$$f_{ij} = B_{ij} *(\delta_i - \delta_j);$$

$$|f_{ij}| \leq f_{ij}^{max};$$

$$0 \leq g \leq g^{\max};$$

where:

C_i the redispatch bid price submitted by Generator i;

ΔP_{i} the active power variation of Generator i;

k the number of generators;

f_{ij} active power flow through Branch i -j;

$f_{ij}^{\max}$ power flow limit in Branch i-j;

B_{ij} total circuit susceptance in Branch i -j;

δ_i , δ_j voltage angle at Buses i and j;

g array of bus active powers;

$g^{\max}$ maximum bus generation capacity array;

Equality constraint $\sum \Delta P_i = 0$ indicates that, for a fixed load demand, the sum of the variations of active outputs of power plants should be equal to zero to keep the system's power balanced.

The proposed optimal power flow method is described as follows:

1) Use DC power flow analyses to obtain the power flow of each transmission line under the original generation schedule. A bus must be selected as reference bus. In order to observe the influence of each generator to the line flow, choose a P-Q bus as reference bus, and thus, the power balance equations can be represented in the form of a matrix:

$$P = B\delta \qquad (3.2)$$

where, P is the vector of bus power injections, B is the bus admittance matrix and δ is the vector of bus angles. Using Equation (3.2), δ can be obtained as:

$$\delta = B^{-1} * P \qquad (3.3)$$

The following expression can be used to get line flows.

$$f_{ij} = B_{ij} * (\delta_i - \delta_j) \qquad (3.4)$$

$|f_{ij}| > f_{ij}^{\max}$ indicates that congestion has occurred in transmission Line ij.

2) From Equation (3.3), the sensitivity matrix of generator outputs to line flows can be obtained in the following manner:

$$\delta = B^{-1} * P = B^{-1} * (P_G + P_D)$$

$$\delta^0 + \Delta\delta = B^{-1} * (P^0 + \Delta P)$$

$$= B^{-1} * (P_G^0 + \Delta P_G) + B^{-1} * (P_D^0 + \Delta P_D)$$

where P_G is the vector of active output of power plants, P_D is the vector of load demand, δ^0, P_G^0 and P_D^0 are the initial vectors and $\Delta\delta$, ΔP_G and ΔP_D are the variation vectors. Because the load demand is fixed, $\Delta P_D = 0$. The sensitivity expressions can be written as:

$$\delta^0 + \Delta\delta = B^{-1} * (P_G^0 + \Delta P_G) + B^{-1} * P_D^0$$

$$= B^{-1} * (P_G^0 + P_D^0) + B^{-1} * \Delta P_G$$

Since $\delta^0 = B^{-1} * (P_G^0 + P_D^0)$,

$$\Delta\delta = B^{-1} * \Delta P_G \qquad (3.5)$$

Let M denote the line-bus incidence matrix. Also, let Y be the diagonal matrix of negative line susceptances and let F be the vector of real power flows. Then the vector of active power flows can be written as:

$$F = Y^*M^*\delta \tag{3.6}$$

If ΔF is the vector of the variations of active power flows, then:

$$F^0 + \Delta F = Y^*M^*(\delta^0 + \Delta\delta)$$

$$= Y^*M^*\delta^0 + Y^*M^*\Delta\delta$$

Because $F^0 = Y^*M^*\delta^0$, therefore:

$$\Delta F = Y^*M^*\Delta\delta$$

$$= Y^*M^* B^{-1} {}^*\Delta P_G \tag{3.7}$$

$$= S^*\Delta P_G$$

$$S = Y^*M^* B^{-1}$$

S is the sensitivity matrix.

3) To relieve a congestion, the original generation schedule should be changed. It means that outputs of some generators have to be reduced, and at the same time, outputs of others should be increased. One has to determine which generators should increase their active power, and which generators should decrease their active power. Here, 1 MW is used as a step size to find out the possible schemes to relieve a congestion. For example, assume that transmission Line k is overloaded. Let ΔF_k be the power flow variation of Line k, and $S_{k(i)}$ is the sensitivity coefficient of Generator i to ΔF_k, and there are m generators in the system. ΔF_k can be expressed as:

$$\Delta F_k = S_{k(1)}{}^*\Delta P_{G1} + \ldots + S_{k(i)}{}^*\Delta P_{Gi} + \ldots + S_{k(j)}{}^*\Delta P_{Gj} + \ldots + S_{k(m)}{}^*\Delta P_{Gm}$$

If the output of Generator i is increased by 1MW, and at the same time, the output of Generator j is decreased by 1MW, the power flow variation in Line k can be obtained as:

$$\Delta F_k(i,j) = S_{k(i)} - S_{k(j)} \tag{3.8}$$

In this way, the power flow variation matrix ΔF_k for Line k can be obtained. The element $\Delta F_k(i,j)$ corresponds to the power flow variation in Line k due to an increase of 1MW of output by Generator i and a decrease of 1MW by Generator j. Replace those elements that will increase the power flow in the congested Line k with zeros. If there are several overloaded transmission lines at the same time, their corresponding power flow variation matrices can be formed respectively; and then, keep those elements which will not cause other lines congested and let other elements be replaced with zeros.

4) Form the corresponding incremental cost matrix for each power flow variation matrix. If $\Delta F_k(i,j) \neq 0$, its corresponding incremental cost $\Delta C_k(i,j)$ can be obtained in the following manner:

$$\begin{aligned} \Delta D_k(i,j) &= C_i{*}\Delta P_{\mathrm{Gi}} - C_j{*}\Delta P_{\mathrm{Gi}} \\ &= C_i{*}1 - C_j{*}1 \\ &= C_i - C_j \end{aligned} \tag{3.9}$$

$$\Delta C_k(i,j) = \frac{\Delta D_k(i,j)}{|\Delta F_k(i,j)|} = \frac{C_i - C_j}{|S_{k(i)} - S_{k(j)}|} \tag{3.10}$$

$\Delta D_k(i,j)$ is the generation cost increase due to an increase of 1 MW of output by Generator i and a decrease of 1 MW of output by Generator j.

Element $\Delta C_k(i,j)$ represents the fact that if 1 MW of power flow in transmission Line k is relieved, the generation cost will increase by $\Delta C_k(i,j)$ when only the active outputs of Generators i and j ($\Delta P_{Gi} + \Delta P_{Gj} = 0$) are changed. From the incremental cost matrix ΔC_k, we can easily obtain the most economical way to relieve the congestion of transmission Line k.

5) When the most economical way is determined for each overloaded line, check the combined effect of two or more overloaded lines. Assume that the best way to relieve the congestion in transmission Line k is to increase the active output at Bus i and decrease the output at Bus j. Also assume that the best way to relieve the congestion in transmission Line q is to increase the active output at Bus m and decrease the output at Bus n, then,

$$\Delta P_G = \begin{bmatrix} 0 \\ \cdots \\ \Delta P_{Gi} \\ \cdots \\ \Delta P_{Gj} \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \cdots \\ \Delta P_{Gm} \\ \cdots \\ \Delta P_{Gn} \\ \cdots \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \cdots \\ \Delta P_{Gi} \\ \cdots \\ \Delta P_{Gj} \\ \cdots \\ \Delta P_{Gm} \\ \cdots \\ \Delta P_{Gn} \\ \cdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \cdots \\ 1 \\ \cdots \\ -1 \\ \cdots \\ 1 \\ \cdots \\ -1 \\ \cdots \\ 0 \end{bmatrix}$$

From Equation (3.7), one get:

$$\Delta F = Y^*M^* B^{-1}*\Delta P_G$$

The line flows will become:

$$F^{(k+1)} = F^{(k)} + \Delta F;$$

where, k is the number of iteration.

For the first iteration,

$$F^1 = F^0 + \Delta F;$$

where, F^0 is the initial line flow vector under the original generation schedule. After each iteration, check to see if the power variation, ΔP_G is causing a previously non-congested line overloaded. If such situation happens, replace those corresponding elements in matrices ΔF_k and ΔC_k, which cause this congestion, with zeros. And then, find a new optimum incremental cost matrix ΔC to replace the old one. Generally, most of transmission systems are designed to be robust. Therefore, this situation rarely occurs. If the power variation, ΔP_G is causing a congested line back to normal operating state, only those transmission lines that are still overloaded are considered during the next iteration.

6) After each iteration, we should check the active power output of each generator. If the output reaches the generation limit, set it to the limit value, and then, find a new optimum incremental cost matrix ΔC to replace the old one.

A simple system as shown in Fig. 3.1 is considered to illustrate the steps mentioned above with a numerical example. Three transactions are considered for this

6-bus system as shown in Fig. 3.1. The magnitudes of the transactions and the line flow limits are also indicated in the figure. Transactions T1 and T2 are scheduled to supply the load at Node 3, and transaction T3 is scheduled to supply the load at Node 4.

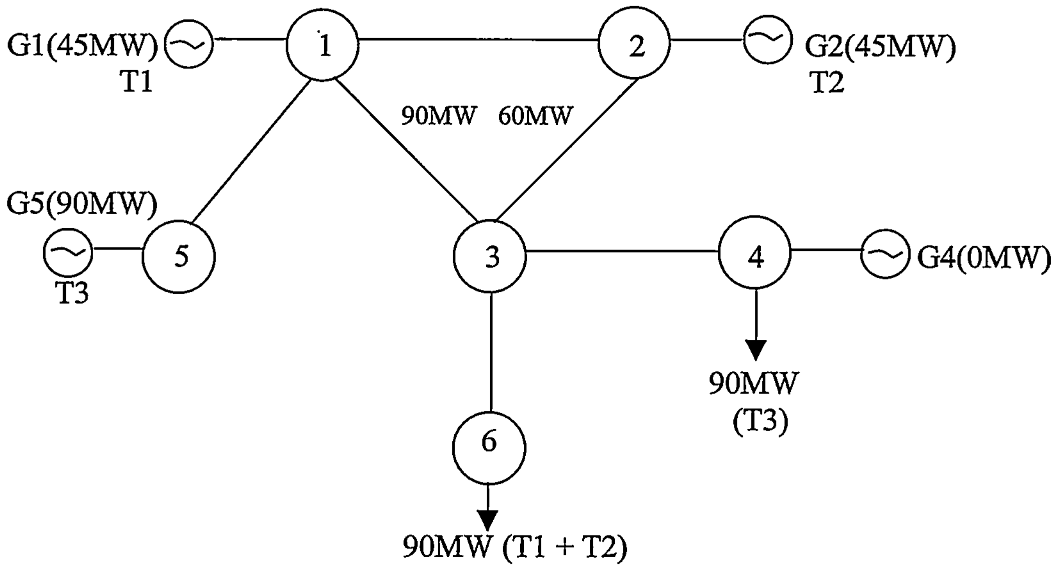


Fig. 3.1 A Simple System with Three Electric Transactions

The Table 3.1 shows a list of the transmission lines:

Table 3.1: List of Transmission Lines

No.	From	To
1	1	2
2	1	3
3	1	5
4	2	3
5	3	4
6	3	6

The system is operated and controlled by an ISO. In order to relieve possible congestion, the generators are required to submit their redispatch prices to the ISO. The generators' redispatch prices are shown in Fig. 3.2. Note that, as indicated in the figure, the price curves have usually two segments, one for increasing, and the other for backing down the generation from the desired operating point.

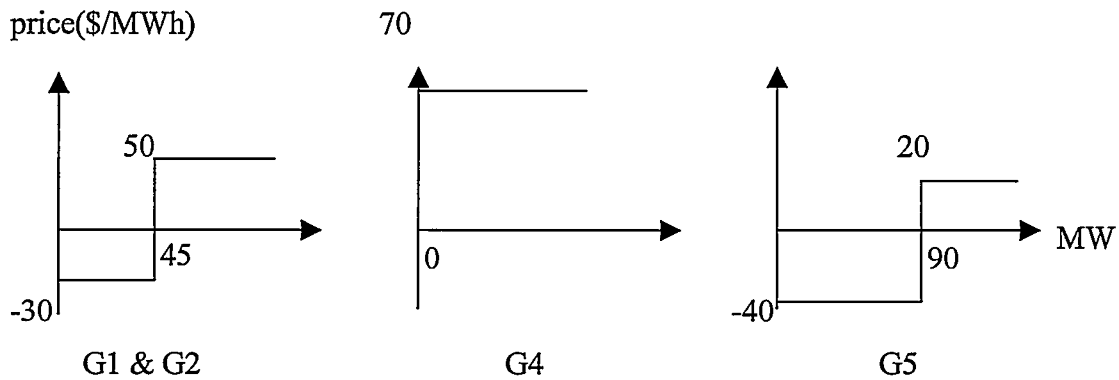


Fig. 3.2 Generation Redispatch Bid Price

The proposed method, as mentioned in Section 3.1.1, has been utilized to determine the amount of redispatched power for each generator. In order to simplify the calculations, all lines have been represented by an identical reactance of 0.1 p.u. Therefore, the admittance of each transmission line is 1/0.1 = 10. Node 6 has been selected as the reference bus. The admittance matrix, N, is:

$$N = \begin{bmatrix} 30 & -10 & -10 & 0 & -10 & 0 \\ -10 & 20 & -10 & 0 & 0 & 0 \\ -10 & -10 & 40 & -10 & 0 & -10 \\ 0 & 0 & -10 & 10 & 0 & 0 \\ -10 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & -10 & 0 & 0 & 10 \end{bmatrix}$$

By eliminating column 6 and row 6 from N, one can get the matrix B as:

$$B = \begin{bmatrix} 30 & -10 & -10 & 0 & -10 \\ -10 & 20 & -10 & 0 & 0 \\ -10 & -10 & 40 & -10 & 0 \\ 0 & 0 & -10 & 10 & 0 \\ -10 & 0 & 0 & 0 & 10 \end{bmatrix}$$

The inverse of B can be written as:

$$B^{-1} = \begin{bmatrix} 0.1667 & 0.1333 & 0.1000 & 0.1000 & 0.1667 \\ 0.1333 & 0.1667 & 0.1000 & 0.1000 & 0.1333 \\ 0.1000 & 0.1000 & 0.1000 & 0.1000 & 0.1000 \\ 0.1000 & 0.1000 & 0.1000 & 0.2000 & 0.1000 \\ 0.1667 & 0.1333 & 0.1000 & 0.1000 & 0.2667 \end{bmatrix}$$

The line-bus incidence matrix, M and the diagonal matrix of negative line suceptances, Y are:

$$M = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix}$$

The initial line flow vector F^0 can be obtained as:

$$F^0 = Y*M*\delta^0 = Y*M*B^{-1}*P^0 = S*P^0$$

$$= \begin{bmatrix} 0.3333 & -0.3333 & 0 & 0 & 0.3333 \\ 0.6667 & 0.3333 & 0 & 0 & 0.6667 \\ 0 & 0 & 0 & 0 & -1.0000 \\ 0.3333 & 0.6667 & 0 & 0 & 0.3333 \\ 0 & 0 & 0 & -1.0000 & 0 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} 45 \\ 45 \\ 0 \\ -90 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 30.0000 \\ 105.0000 \\ -90.0000 \\ 75.0000 \\ 90.0000 \\ 90.0000 \end{bmatrix}$$

The initial line flows indicate that Line 2 (from 2 to 3) and Line 4 (from 2 to 3) are overloaded. The magnitudes of congestion in these two transmission lines are both 15MW. The respective power flow variations of Line 2 and Line 4 can be written as:

$$\Delta F_2 = S_{11}*\Delta P_{G1} + \ldots + S_{1i}*\Delta P_{Gi} + \ldots + S_{1j}*\Delta P_{Gj} + \ldots + S_{1m}*\Delta P_{Gm}$$

$$= 0.6667*\Delta P_{G1} + 0.3333*\Delta P_{G2} + 0*\Delta P_{G4} + 0.6667*\Delta P_{G5}$$

$$\Delta F_4 = 0.3333*\Delta P_{G1} + 0.6667*\Delta P_{G2} + 0*\Delta P_{G4} + 0.3333*\Delta P_{G5}$$

The respective power flow variation matrices $\boldsymbol{\Delta F_2}$ and $\boldsymbol{\Delta F_4}$ for Line 2 and Line 4 can be obtained by using $\Delta F_k(i,j) = S_{k(i)} - S_{k(j)}$:

$$\Delta F_2 = \begin{array}{c} \\ G1\uparrow \\ G2\uparrow \\ G4\uparrow \\ G5\uparrow \end{array} \begin{array}{c} \begin{array}{cccc} G1\downarrow & G2\downarrow & G4\downarrow & G5\downarrow \end{array} \\ \begin{bmatrix} 0 & 0.3334 & 0.6667 & 0 \\ -0.3334 & 0 & 0.3333 & -0.3334 \\ -0.6667 & -0.3333 & 0 & -0.6667 \\ 0 & 0.3334 & 0.6667 & 0 \end{bmatrix} \end{array}$$

The negative values in $\boldsymbol{\Delta F_2}$, for example, $\boldsymbol{\Delta F_2}(2,1) = -0.3334$, mean that if the active power output of Generator 2 is increased by 1MW and the active power output of Generator 1 is decreased by 1MW, the power flow in Line 2 will decrease by 0.3334 MW. The congestion in Line 2 will be alleviated. A positive value indicates that it will cause an increase of power flow in Line 2. Therefore, only the negative values are kept and the positive values are replaced with zeros. In this way, the power flow variation matrix, $\boldsymbol{\Delta F_2}$ becomes:

$$\Delta F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.3334 & 0 & 0 & -0.3334 \\ -0.6667 & -0.3333 & 0 & -0.6667 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the same procedure, $\boldsymbol{\Delta F_4}$ can be obtained as well:

$$\Delta F_4 = \begin{bmatrix} 0 & -0.3334 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.3333 & -0.6667 & 0 & -0.3333 \\ 0 & -0.3334 & 0 & 0 \end{bmatrix}$$

The next step is to compare the corresponding elements in both power flow variation matrices to find out if an element represents an increase in flow in one line even though its corresponding element represents a decrease in flow in the other line. For instance, $\boldsymbol{\Delta F_2}(2,1) = -0.3334$, and the corresponding element in $\boldsymbol{\Delta F_4}$, $\boldsymbol{\Delta F_4}(2,1) = 0$.

This indicates that even though an increase of active power output of Generator 2 by 1MW and a decrease of active power output of Generator 1 by 1MW would decrease the power flow in Line 2, the same action would increase the power flow in Line 4. Obviously, this action will make Line 4 overloaded. Therefore, $\Delta F_2(2,1)$ is replaced with a zero. Thus, the matrices ΔF_2 and ΔF_4, become:

$$\Delta F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.6667 & -0.3333 & 0 & -0.6667 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta F_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.3333 & -0.6667 & 0 & -0.3333 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The corresponding incremental cost matrices ΔC_2 and ΔC_4 can be obtained by using Equation (3.9) and Equation (3.10):

$$\Delta D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 70\text{-}30 & 70\text{-}30 & 0 & 70\text{-}40 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 40 & 40 & 0 & 30 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 40/0.6667 & 40/0.3333 & 0 & 30/0.6667 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 60 & 120 & 0 & 45 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Delta C_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 40/0.3333 & 40/0.6667 & 0 & 30/0.3333 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 120 & 60 & 0 & 90 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From matrices ΔC_2 and ΔC_4, it is observed that the best way to relieve the congestion in Line 2 is to increase the output of Generator 4 and decrease the output of Generator 5. The incremental cost for this redispacth is \$45/hr for relieving 1MW from Line 2. For Line 4, the best way is to increase the output of Generator 4 and decrease the output of Generator 2. The incremental cost for this redispacth is \$60/hr. Therefore, ΔP_G becomes:

$$\Delta P_G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

$$\Delta F = Y^*M^*B^{-1}{}^*\Delta P_G = S^*\Delta P_G$$

$$= \begin{bmatrix} 0.3333 & -0.3333 & 0 & 0 & 0.3333 \\ 0.6667 & 0.3333 & 0 & 0 & 0.6667 \\ 0 & 0 & 0 & 0 & -1.0000 \\ 0.3333 & 0.6667 & 0 & 0 & 0.3333 \\ 0 & 0 & 0 & -1.0000 & 0 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0000 \\ -1.0000 \\ 1.0000 \\ -1.0000 \\ -2.0000 \\ 0.0000 \end{bmatrix}$$

$$F^1 = F^0 + \Delta F$$

$$= \begin{bmatrix} 30.0000 \\ 105.0000 \\ -90.0000 \\ 75.0000 \\ 90.0000 \\ 90.0 \end{bmatrix} + \begin{bmatrix} 0.0000 \\ -1.0000 \\ 1.0000 \\ -1.0000 \\ -2.0000 \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 30.0000 \\ 104.0000 \\ -89.0000 \\ 74.0000 \\ 88.0000 \\ 90.0000 \end{bmatrix}$$

From the resulting line flows after 1st iteration, it is noticed that power flows in both Line 2 and Line 4 drop by 1 MW. The final results of redispatch for this simple system are that Generator 2 drops 15 MW, Generator 5 drops 15MW and Generator 4 increases 30 MW. These results are the same as those reported in Reference [20].

The total cost of the redispatch for above-mentioned congestion is:

$$C_t = C_2 * \Delta P_{G2} + C_4 * \Delta P_{G4} + C_5 * \Delta P_{G5}$$

$$= -30*15 + 70*30 + (-40)*15 = \$1050/\text{hr}$$

3.1.2 Numerical Example of DC Method Application

The system shown in Fig. 3.1 is used to numerically illustrate the evaluation of annual congestion fee. The daily load demands for T1, T2 and T3 are listed in Table 3.2. The transmission limit of each line is the same as shown that in Fig. 3.1.

Table 3.2: Daily Load Demand by Each Transaction (MW)

Hr / T	1	2	3	4	5	6	7	8	9	10	11	12
T1	24	24	24	24	30	30	30	30	45	45	54	54
T2	24	24	24	24	30	30	30	30	45	45	54	54
T3	60	60	60	60	75	75	75	75	90	90	90	90

Hr / T	13	14	15	16	17	18	19	20	21	22	23	24
T1	54	54	54	54	45	45	45	30	30	30	24	24
T2	54	54	54	54	45	45	45	30	30	30	24	24
T3	90	90	90	90	90	90	90	75	75	75	60	60

Using load demand data shown in Table 3.2 as original generation schedule, the power flow in each transmission line can be obtained. The power flows in the transmission lines are shown in Table 3.3.

Table 3.3: Power Flow in Each Transmission Line (MW)

L \ Hr	1	2	3	4	5	6	7	8	9	10	11	12
1	20	20	20	20	25	25	25	25	30	30	35	35
2	64	64	64	64	80	80	80	80	**105**	**105**	**114**	**114**
3	-60	-60	-60	-60	-75	-75	-75	-75	-90	-90	-105	-105
4	44	44	44	44	55	55	55	55	**75**	**75**	**84**	**84**
5	60	60	60	60	75	75	75	75	90	90	120	120
6	48	48	48	48	60	60	60	60	90	90	105	105

L \ Hr	13	14	15	16	17	18	19	20	21	22	23	24
1	35	35	35	35	30	30	30	25	25	25	20	20
2	**114**	**114**	**114**	**114**	**105**	**105**	**105**	80	80	80	64	64
3	-105	-105	-105	-105	-90	-90	-90	-75	-75	-75	-60	-60
4	**84**	**84**	**84**	**84**	**75**	**75**	**75**	55	55	55	44	44
5	120	120	120	120	90	90	90	75	75	75	60	60
6	105	105	105	105	90	90	90	60	60	60	48	48

It can be noticed from Table 3.3 that congestion occurs in Lines 2 and 4 from 9:00 – 19:00 hours. Therefore, the ISO has to eliminate the congestion. Employing the proposed optimal power flow programming method, a redispatch schedule for each generator can be obtained. The redispatch schedule and the corresponding output of the generators are shown in Table 3.4 and 3.5 in a respective manner:

Table 3.4: Generation Redispatch (MW)

G \ Hr	1	2	3	4	5	6	7	8	9	10	11	12
G1	0	0	0	0	0	0	0	0	0	0	0	0
G2	0	0	0	0	0	0	0	0	-15	-15	-24	-24
G4	0	0	0	0	0	0	0	0	30	30	48	48
G5	0	0	0	0	0	0	0	0	-15	-15	-24	-24

G \ Hr	13	14	15	16	17	18	19	20	21	22	23	24
G1	0	0	0	0	0	0	0	0	0	0	0	0
G2	-24	-24	-24	-24	-15	-15	-15	0	0	0	0	0
G4	48	48	48	48	30	30	30	0	0	0	0	0
G5	-24	-24	-24	-24	-15	-15	-15	0	0	0	0	0

Table 3.5: Actual Active Power Output of Each Generator (MW)

G \ Hr	1	2	3	4	5	6	7	8	9	10	11	12
G1	24	24	24	24	30	30	30	30	45	45	54	54
G2	24	24	24	24	30	30	30	30	30	30	30	30
G4	0	0	0	0	0	0	0	0	30	30	48	48
G5	60	60	60	60	75	75	75	75	75	75	66	66

G \ Hr	13	14	15	16	17	18	19	20	21	22	23	24
G1	54	54	54	54	45	45	45	0	0	0	0	0
G2	30	30	30	30	30	30	30	0	0	0	0	0
G4	48	48	48	48	30	30	30	0	0	0	0	0
G5	66	66	66	66	75	75	75	0	0	0	0	0

Based on the redispatch schedule, the hourly congestion fee for the whole system can be computed.

Table 3.6: Hourly Congestion Fee ($/Hr)

G \ Hr	1	2	3	4	5	6	7	8	9	10	11	12
G1	0	0	0	0	0	0	0	0	0	0	0	0
G2	0	0	0	0	0	0	0	0	-450	-450	-720	-720
G4	0	0	0	0	0	0	0	0	2100	2100	3360	3360
G5	0	0	0	0	0	0	0	0	-600	-600	-960	-960
Total	0	0	0	0	0	0	0	0	1050	1050	1680	1680
G \ Hr	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
G1	0	0	0	0	0	0	0	0	0	0	0	0
G2	-720	-720	-720	-720	-450	-450	-450	0	0	0	0	0
G4	3360	3360	3360	3360	2100	2100	2100	0	0	0	0	0
G5	-960	-960	-960	-960	-600	-600	-600	0	0	0	0	0
Total	1680	1680	1680	1680	1050	1050	1050	0	0	0	0	0

The total congestion fee for one day is:

$$1050*5 + 1680*6 = \$15{,}330/\text{per day}$$

The total congestion fee for one year is:

$$\$15{,}330/\text{per day} * 365 = \$5{,}595{,}450/\text{yr}$$

Obviously, this is a heavy extra cost that will be allocated among the customers. In this case, the ISO should consider the feasibility of building new transmission lines to relieve the congestion and decrease the customers' financial burden. For the ISO, the question is which expansion plan is the best. The ISO should find out the optimal expansion scheme.

In order to show the search process for finding an optimal expansion scheme, the following yearly cost (shown in Table 3.7) of connection is assumed:

Table 3.7: Yearly Cost of All Possible Connections

Possible Connection	Buses From	to	X (p.u.)	Cost ($/per year)
1	1	2	0.10	500,000
2	1	3	0.10	500,000
3	1	4	0.18	900,000
4	1	5	0.10	500,000
5	1	6	0.16	800,000
6	2	3	0.10	500,000
7	2	4	0.14	800,000
8	2	5	0.19	950,000
9	2	6	0.17	850,000
10	3	4	0.10	500,000
11	3	5	0.11	550,000
12	3	6	0.10	500,000
13	4	5	0. 20	1,000,000
14	4	6	0.15	750,000
15	5	6	0.16	800,000

Table 3.7 shows that there are 15 possible connections in the system shown in Fig. 3.1. But different connections will produce different results. Because the congestion occurs during 9:00 to 19:00 hours, only the power flows at these congestion hours are calculated. In order to simplify the evaluation, one can assume that the investment cost is proportional to the length of transmission line. It is not necessary to make this assumption in practice. Actually, investment cost is not only related to the length, but also related to many other factors such as voltage grade, land price and construction difficulties. But for any candidate transmission line, the best construction scheme, which will bring the lowest investment cost, can be determined. And these costs can be considered as the investment costs of all possible connections. Table 3.8 shows the results of power flow changes in the congested transmission lines. In order to assess the economic impact of adding a connection, evaluation is done for each possible connection.

Table 3.8: Power Flow Changes in Congested Lines (MW)

Possible Connection	Congested Line	9:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00
1	2	99	99	108	108	108	108	108	108	99	99	99
	4	81	81	90	90	90	90	90	90	81	81	81
2	2	63	63	68	68	68	68	68	68	63	63	63
	4	54	54	61.2	61.2	61.2	61.2	61.2	61.2	54	54	54
3	2	67.5	67.5	74.8	74.8	74.8	74.8	74.8	74.8	67.5	67.5	67.5
	4	56.3	56.3	64.4	64.4	64.4	64.4	64.4	64.4	56.3	56.3	56.3
4	2	105	105	114	114	114	114	114	114	105	105	105
	4	75	75	84	84	84	84	84	84	75	75	75
5	2	65.2	65.2	68.7	68.7	68.7	68.7	68.7	68.7	65.2	65.2	65.2
	4	55.1	55.1	61.4	61.4	61.4	61.4	61.4	61.4	55.1	55.1	55.1
6	2	90	90	97.2	97.2	97.2	97.2	97.2	97.2	90	90	90
	4	45	45	50.4	50.4	50.4	50.4	50.4	50.4	45	45	45
7	2	87.1	87.1	95.1	95.1	95.1	95.1	95.1	95.1	87.1	87.1	87.1
	4	39.1	39.1	46.2	46.2	46.2	46.2	46.2	46.2	39.1	39.1	39.1
8	2	93.8	93.8	102.8	102.8	102.8	102.8	102.8	102.8	93.8	93.8	93.8
	4	86.2	86.2	95.2	95.2	95.2	95.2	95.2	95.2	86.2	86.2	86.2
9	2	88.7	88.7	95	95	95	95	95	95	88.7	88.7	88.7
	4	42.3	42.3	46	46	46	46	46	46	42.3	42.3	42.3
10	2	105	105	114	114	114	114	114	114	105	105	105
	4	75	75	84	84	84	84	84	84	75	75	75
11	2	58	58	64.8	64.8	64.8	64.8	64.8	64.8	58	58	58
	4	51.5	51.5	59.4	59.4	59.4	59.4	59.4	59.4	51.5	51.5	51.5
12	2	105	105	114	114	114	114	114	114	105	105	105
	4	75	75	84	84	84	84	84	84	75	75	75
13	2	64.3	64.3	72	72	72	72	72	72	64.3	64.3	64.3
	4	54.6	54.6	63	63	63	63	63	63	54.6	54.6	54.6
14	2	105	105	114	114	114	114	114	114	105	105	105
	4	75	75	84	84	84	84	84	84	75	75	75
15	2	60.5	60.5	65.3	65.3	65.3	65.3	65.3	65.3	60.5	60.5	60.5
	4	52.7	52.7	59.6	59.6	59.6	59.6	59.6	59.6	52.7	52.7	52.7

In Table 3.8, bold numbers represent congestion. From the results shown in Table 3.8, one can notice that, although congestion occurs in Line 2 (from Bus1 to Bus 3) and Line 4 (from Bus 2 to Bus 3), it is not necessary to build two transmission lines in parallel with the congested lines to relieve the congestion. In this simple system, if it operates under the specified transactions, building one transmission line is enough to solve the congestion problem. There are two solutions to relieve the congestion. One is to build a transmission line between Bus 3 and Bus 5. Another solution is adding a transmission line between Bus 5 and Bus 6. The possible solutions are shown in Fig. 3.3 by dotted lines.

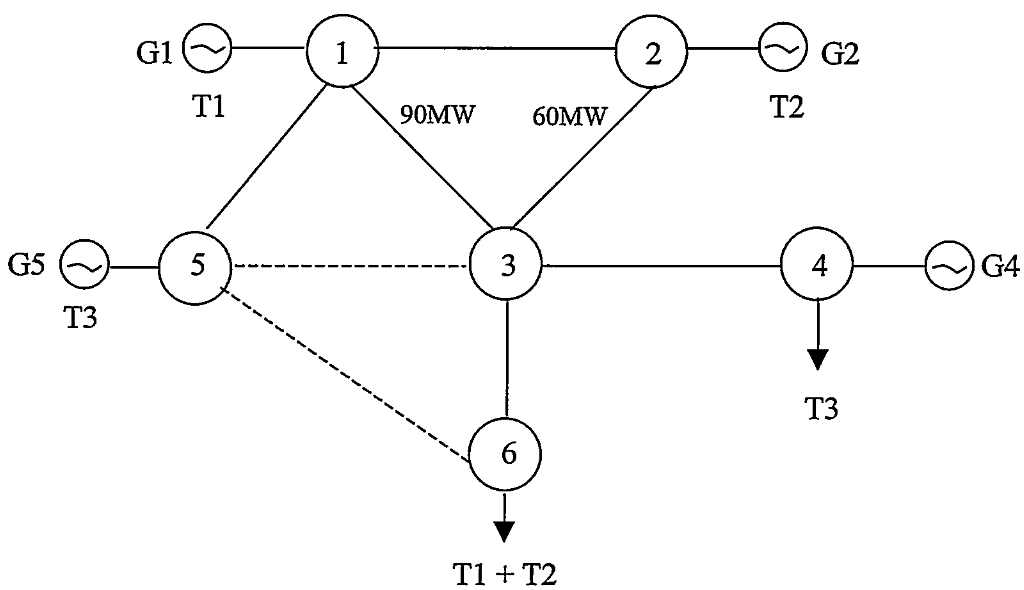


Fig. 3.3 Two Possible Solutions for Relieving System Congestion

It can be noticed from the results tabulated in Table 3.8 that these two solutions will bring almost the same benefit to the system. But the investment costs for building these two transmission lines are different. The yearly cost for the new transmission line between Bus 3 and Bus 5 is $550,000 dollars. The other line between Bus 5 and Bus 6 will cost $800,000 dollars. Between these two, the first one will save about $250,000/yr compared to the second one. If the first one is chosen as our expansion plan, the benefit brought to the customers will be:

$$\$5{,}595{,}450/\text{yr} - \$550{,}000/\text{yr} = \$5{,}045{,}450/\text{yr}$$

The economic benefit dictates that the ISO should build this transmission line. In addition, this new line will bring another benefit to the system. From Fig. 3.3, one can see that Generator 5 will have two transmission lines available to transmit its power to the system. This will increase the overall stability and reliability of the system. From the viewpoint of the customers, this new line will save a large amount of money for them.

3.2 AC Optimal Power Flow Programming Method

Although DC optimal power flow programming method described in the previous section is very simple, it doesn't take transmission losses into account. This is a major disadvantage of the DC optimal power flow method. Consideration of transmission loss is very important when a system operation mode is arranged, because, it is tightly related to the operation cost of a system. Therefore, it is desirable to find a method that takes transmission losses into consideration. A proposed programming method called AC method that includes transmission loss is introduced in this section. This method is based on the following assumptions:

1) Considering that the redispatch of generation is actually exercised among a few generators when congestion happens, its impact on the voltage of the whole system could be neglected. Therefore, it is assumed that the voltage is invariant during the process of evaluation.

2) Redispatching of only the active power outputs of the related generators are considered. The impact brought by the change of reactive output of each generator is very small to the distribution of active power flow and hence can be neglected.
3) A relatively small increment is utilized in each iteration, therefore, a linear approximation can be used to treat the non-linear problem.

3.2.1 Mathematical Development of Transmission Loss Formula

Although Kron's transmission loss Formula is very convenient in describing the relationship between the active power output of each generator and the transmission loss, it is valid only for a certain range of system operation pattern due to a number of approximations involved in its formation such as constant bus angle δ_i and fixed load pattern. Its accuracy is not satisfactory if system operation pattern has a large change [21]. A more accurate transmission loss formula [22] that takes account of bus angle δ_i and bus injection power P_i is explained below.

Transmission loss in a system is equal to the sum of the powers injected into each node:

$$S_L = P_L + jQ_L = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} \boldsymbol{U}_i \boldsymbol{I}_i^*$$

where,

S_L the total complex power loss

P_L the total active power loss

Q_L the total reactive power loss

S_i the complex power injected into node i

$\boldsymbol{U}_i$ the voltage phasor of node i

$\boldsymbol{I}_i^*$ the conjugate of the current phasor injected into node i

The above expression can be written in the form of a matrix equation:

$$P_L + jQ_L = \boldsymbol{U}_B^t \boldsymbol{I}_B^*$$

where,

$\boldsymbol{U}_B$ the vector of bus voltage

$\boldsymbol{I}_B$ the vector of current injected into bus

Let $\boldsymbol{Z}_B$ be the bus impedance matrix. As $\boldsymbol{U}_B = \boldsymbol{Z}_B \boldsymbol{I}_B$, and $\boldsymbol{Z}_B$ is a symmetric matrix, one will get:

$$P_L + jQ_L = \boldsymbol{I}_B^t \boldsymbol{Z}_B \boldsymbol{I}_B^* \qquad (3.11)$$

Decomposing $\boldsymbol{Z}_B$ into resistance part and reactance part and also decomposing $\boldsymbol{I}_B$ into active part and reactive part, one can get:

$$\boldsymbol{Z}_B = \boldsymbol{R} + j^*\boldsymbol{X} = \begin{bmatrix} R_{11} & R_{12} & \cdots\cdots & R_{1n} \\ R_{21} & R_{22} & \cdots\cdots & R_{2n} \\ . & . & \cdots\cdots & . \\ . & . & \cdots\cdots & . \\ R_{n1} & R_{n2} & \cdots\cdots & R_{nn} \end{bmatrix} + j \begin{bmatrix} X_{11} & X_{12} & \cdots\cdots & X_{1n} \\ X_{21} & X_{22} & \cdots\cdots & X_{2n} \\ . & . & \cdots\cdots & . \\ . & . & \cdots\cdots & . \\ X_{n1} & X_{n2} & \cdots\cdots & X_{nn} \end{bmatrix}$$

$$\boldsymbol{I}_B = \boldsymbol{I}_p + j^*\boldsymbol{I}_q = \begin{bmatrix} I_{p1} \\ I_{p2} \\ . \\ . \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ . \\ . \\ I_{qn} \end{bmatrix}$$

Substituting above expressions into Equation (3.11):

$$P_L + jQ_L = \boldsymbol{I}_B^t \boldsymbol{Z}_B \boldsymbol{I}_B^* = (\boldsymbol{I}_p + j\boldsymbol{I}_q)^t(\boldsymbol{R} + j\boldsymbol{X})(\boldsymbol{I}_p - j\boldsymbol{I}_q)$$

$$= (\boldsymbol{I}_p^t \boldsymbol{R} \boldsymbol{I}_p + \boldsymbol{I}_q^t \boldsymbol{R} \boldsymbol{I}_q + \boldsymbol{I}_p^t \boldsymbol{R} \boldsymbol{I}_q - \boldsymbol{I}_q^t \boldsymbol{R} \boldsymbol{I}_p)$$

$$+ j(\boldsymbol{I}_p^t \boldsymbol{X} \boldsymbol{I}_p + \boldsymbol{I}_q^t \boldsymbol{X} \boldsymbol{I}_q + \boldsymbol{I}_q^t \boldsymbol{X} \boldsymbol{I}_p - \boldsymbol{I}_p^t \boldsymbol{X} \boldsymbol{I}_q)$$

Equating the real and imaginary part:

$$P_L = \boldsymbol{I}_p^t \boldsymbol{R} \boldsymbol{I}_p + \boldsymbol{I}_q^t \boldsymbol{R} \boldsymbol{I}_q + \boldsymbol{I}_p^t \boldsymbol{X} \boldsymbol{I}_q - \boldsymbol{I}_q^t \boldsymbol{X} \boldsymbol{I}_p$$

$$Q_L = \boldsymbol{I}_p^t\boldsymbol{X}\boldsymbol{I}_p + \boldsymbol{I}_q^t\boldsymbol{X}\boldsymbol{I}_q + \boldsymbol{I}_q^t\boldsymbol{R}\boldsymbol{I}_p - \boldsymbol{I}_p^t\boldsymbol{R}\boldsymbol{I}_q$$

Due to the symmetry of matrices **R** and **X**:

$$\boldsymbol{I}_p^t\boldsymbol{X}\boldsymbol{I}_q - \boldsymbol{I}_q^t\boldsymbol{X}\boldsymbol{I}_p = 0$$

$$\boldsymbol{I}_p^t\boldsymbol{R}\boldsymbol{I}_q - \boldsymbol{I}_q^t\boldsymbol{R}\boldsymbol{I}_p = 0$$

Therefore,

$$P_L = \boldsymbol{I}_p^t\boldsymbol{R}\boldsymbol{I}_p + \boldsymbol{I}_q^t\boldsymbol{R}\boldsymbol{I}_q$$

$$= \sum_{j=1}^{n}\sum_{k=1}^{n} R_{jk}(I_{pj}I_{pk} + I_{qj}I_{qk}) \tag{3.12}$$

$$Q_L = \boldsymbol{I}_p^t\,\boldsymbol{X}\,\boldsymbol{I}_p + \boldsymbol{I}_q^t\,\boldsymbol{X}\,\boldsymbol{I}_q$$

$$= \sum_{j=1}^{n}\sum_{k=1}^{n} X_{jk}\,(I_{pj}{*}I_{pk} + I_{qj}{*}I_{qk}) \tag{3.13}$$

The complex power injection at Bus i can be written as:

$$P_i + jQ_i = \boldsymbol{U}_i\boldsymbol{I}_i^* = \boldsymbol{U}_i(I_{pi} - jI_{qi}) = |\boldsymbol{U}_i|(\cos\delta_i + j\sin\delta_i)\,(I_{pi} - jI_{qi}) \tag{3.14}$$

Equating the real and imaginary parts of Equation (3.2.1.4):

$$P_i = |\boldsymbol{U}_i|(I_{pi}\cos\delta_i + I_{qi}\sin\delta_i) \tag{3.15}$$

$$Q_i = |\boldsymbol{U}_i|(I_{pi}\sin\delta_i - I_{qi}\cos\delta_i) \tag{3.16}$$

Solving Equations (3.15) and (3.16) for I_{pi} and I_{qi}:

$$I_{pi} = (1/|\boldsymbol{U}_i|)(P_i\cos\delta_i + Q_i\sin\delta_i)$$

$$I_{qi} = (1/|\boldsymbol{U}_i|)(P_i\sin\delta_i - Q_i\cos\delta_i)$$

Substitute the above two expressions into Equations (3.12) and (3.13):

$$P_L = \sum_{j=1}^{n}\sum_{k=1}^{n}[\alpha_{jk}(P_jP_k + Q_jQ_k) + \beta_{jk}(Q_jP_k - P_jQ_k)]$$

$$Q_L = \sum_{j=1}^{n}\sum_{k=1}^{n}[\gamma_{jk}(P_jP_k + Q_jQ_k) + \theta_{jk}(Q_jP_k - P_jQ_k)]$$

where,

$$\alpha_{jk} = \frac{R_{jk}}{|\boldsymbol{U}_j||\boldsymbol{U}_k|}\cos(\delta_j - \delta_k),$$

$$\beta_{jk} = \frac{R_{jk}}{|\boldsymbol{U}_j||\boldsymbol{U}_k|}\sin(\delta_j - \delta_k),$$

$$\gamma_{jk} = \frac{X_{jk}}{|\boldsymbol{U}_j||\boldsymbol{U}_k|}\cos(\delta_j - \delta_k) \text{ and}$$

$$\theta_{jk} = \frac{X_{jk}}{|\boldsymbol{U}_j||\boldsymbol{U}_k|}\sin(\delta_j - \delta_k).$$

Let $P_{\mathrm{G}i}$ be the active generation at Node i, and $P_{\mathrm{D}i}$ be the active load demand at Node i, then, the net active power injected into Node i is:

$$P_i = P_{\mathrm{G}i} - P_{\mathrm{D}i}$$

$P_{\mathrm{D}i}$ is invariant during redispatching of the generation when congestion happens. Therefore, one can get:

$$\partial P_i = \partial P_{\mathrm{G}i} - \partial P_{\mathrm{D}i} = \partial P_{\mathrm{G}i}$$

Then, one will have:

$$\partial P_{\mathrm{L}}/\partial P_{\mathrm{G}i} = \sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial}{\partial P_i}[\alpha_{jk}(P_jP_k + Q_jQ_k) + \beta_{jk}(Q_jP_k - P_jQ_k)] \quad (3.17)$$

The partial derivative of each term in Equation (3.17) is listed in Table 3.9:

Table 3.9: Partial Derivative of Each Term in Equation (3.17)

j	k	$(\partial/\partial P_i)(\alpha_{jk}P_jP_k)$	$(\partial/\partial P_i)(\alpha_{jk}Q_jQ_k)$	$(\partial/\partial P_i)(\beta_{jk}Q_jP_k)$	$(\partial/\partial P_i)(-\beta_{jk}P_jQ_k)$
j=i	k=i	$2P_i\alpha_{ii}+P_i^2(\partial\alpha_{ii}/\partial P_i)$	$Q_i^2(\partial\alpha_{ii}/\partial P_i)$	0	0
j=i	k≠i	$P_k[\alpha_{ik}+P_i(\partial\alpha_{ik}/\partial P_i)]$	$Q_jQ_k(\partial\alpha_{ik}/\partial P_i)$	$Q_jP_k(\partial\beta_{ik}/\partial P_i)$	$-Q_k[\beta_{ik}+P_i(\partial\beta_{ik}/\partial P_i)]$
j≠i	k=i	$P_j[\alpha_{ji}+P_i(\partial\alpha_{ji}/\partial P_i)]$	$Q_jQ_i(\partial\alpha_{ji}/\partial P_i)$	$Q_j[\beta_{ji}+P_i(\partial\beta_{ji}/\partial P_i)]$	$-P_jQ_i(\partial\beta_{ji}/\partial P_i)$
j≠i	k≠i	$P_jP_k(\partial\alpha_{jk}/\partial P_i)$	$Q_jQ_k(\partial\alpha_{jk}/\partial P_i)$	$Q_jP_k(\partial\beta_{jk}/\partial P_i)$	$-P_jQ_k(\partial\beta_{jk}/\partial P_i)$

$$\partial P_L/\partial P_{Gi} = 2\alpha_{ii}P_i + P_i^2(\partial\alpha_{ii}/\partial P_i) + Q_i^2(\partial\alpha_{ii}/\partial P_i)$$

$$+ \sum_{\substack{k=1\\k\neq i}}^{n}[\alpha_{ik}P_k - \beta_{ik}Q_k + (P_iP_k + Q_iQ_k)(\partial\alpha_{ik}/\partial P_i) + (Q_iP_k - P_iQ_k)(\partial\beta_{ik}/\partial P_i)]$$

$$+ \sum_{\substack{j=1\\j\neq I}}^{n}[\alpha_{ji}P_j + \beta_{ji}Q_j + (P_iP_j + Q_iQ_j)(\partial\alpha_{ji}/\partial P_i) + (Q_jP_i - P_jQ_i)(\partial\beta_{ji}/\partial P_i)]$$

$$+ \sum_{\substack{j=1\\j\neq i}}^{n}\sum_{\substack{k=1\\k\neq i}}^{n}[(P_jP_k + Q_jQ_k)(\partial\alpha_{jk}/\partial P_i) + (Q_jP_k - P_jQ_k)(\partial\beta_{jk}/\partial P_i)] \quad (3.18)$$

Because $\alpha_{jk} = \alpha_{kj}$ and $\beta_{jk} = -\beta_{kj}$, Equation (3.18) could be simplified as:

$$\partial P_L/\partial P_{Gi} = 2\sum_{k=1}^{n}(\alpha_{ik}P_k - \beta_{ik}Q_k)$$

$$+ \sum_{j=1}^{n}\sum_{k=1}^{n}[(P_jP_k + Q_jQ_k)(\partial\alpha_{jk}/\partial P_i) - (Q_jP_k - P_jQ_k)(\partial\beta_{jk}/\partial P_i)] \quad (3.19)$$

Because the second part in the right side of Equation (3.19) is generally very small, it can be neglected. Therefore, Equation (3.19*)* could be written as:

$$\partial P_L/\partial P_{Gi} = 2\sum_{k=1}^{n}(\alpha_{ik}P_k - \beta_{ik}Q_k) \quad (3.20)$$

Generally, β_{ik} is much smaller than α_{ik}, and also, Q_k is smaller than P_k. Therefore, $\beta_{ik}Q_k$ can be neglected. Equation (3.20) can be further simplified as:

$$\partial P_L/\partial P_{Gi} = 2\sum_{k=1}^{n}\alpha_{ik}P_k \quad (3.21)$$

3.2.2 Mathematical Development of AC Method

The power balance equation with transmission loss is:

$$\sum_{i=1}^{n}P_{Gi} + \sum_{i=1}^{n}P_{Di} - P_L = 0$$

When we redispatch the generation to relieve congestion, the load remains invariant. Therefore,

$$\sum_{i=1}^{n} \Delta P_{Gi} + \sum_{i=1}^{n} \Delta P_{Di} - \Delta P_L$$

$$= \sum_{i=1}^{n} \Delta P_{Gi} - \Delta P_L = 0$$

The mathematical model for optimizing the redispatched generation to relieve congestion can be described as follows:

$$Min \sum_{i=1}^{n} C_i * \Delta P_{Gi} \quad (3.22)$$

Subject to:

$$\sum_{i=1}^{n} \Delta P_{Gi} - \Delta P_L = 0;$$

$$f_{ij} = B_{ij} * (\delta_i - \delta_j);$$

$$|f_{ij}| \leq f_{ij}^{max};$$

$$0 \leq g \leq g^{max};$$

where

C_i the redispatch price submitted by Generator i;

ΔP_{Gi} the active power variation of Generator i;

n number of generators;

f_{ij} active power flow through Branch i -j;

f_{ij}^{max} flow limit in Branch i-j;

B_{ij} total circuit susceptance in the Branch i -j;

δ_i , δ_j voltage angle at Buses i and j;

g array of bus active powers;

g^{max} maximum bus generation capacity array;

Equality constraint $\Sigma \Delta P_{Gi}$ - $\Delta P_L = 0$ indicates that, for a fixed load demand, the sum of the variations of active outputs of power plants should be equal to the variation of transmission loss.

The power balance equation of Node i is:

$$(P_i + jQ_i) - |U_i| e^{j\delta_i} \sum_{j=1}^{n} (G_{ij} - jB_{ij}) |U_j| e^{-j\delta_j} = 0; \tag{3.23}$$

The real and imaginary parts of Equation (3.2.2.2) can be written as:

$$P_i - |U_i| \sum_{j=1}^{n} |U_j| (G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) = 0; \tag{3.24}$$

$$Q_i - |U_i| \sum_{j=1}^{n} |U_j| (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) = 0; \tag{3.25}$$

where, $\delta_{ij} = \delta_i - \delta_j$.

Let ΔP_i be the increment of active power at Node i, and assume P_i^0 is the initial active power at Node i for a base load flow. The active power at Node i is:

$$\begin{aligned} P_i &= P_i^0 + \Delta P_i \\ &= P_i^0 + \nabla \Delta X \\ &= P_i^0 + J\Delta\delta + H\Delta U \end{aligned} \tag{3.26}$$

where

$$\nabla = [\partial P_i/\partial \delta_1,\ \partial P_i/\partial \delta_2,\ \ldots,\ \partial P_i/\partial \delta_n,\ \partial P_i/\partial U_1,\ \partial P_i/\partial U_2,\ \ldots,\ \partial P_i/\partial U_n]$$

$$\Delta X = \mathrm{col}[\Delta\delta_1,\ \Delta\delta_2,\ \ldots,\ \Delta\delta_n,\ \Delta U_1,\ \Delta U_2,\ \ldots,\ \Delta U_n]$$

$$J = [\partial P_i/\partial \delta_1,\ \partial P_i/\partial \delta_2,\ \ldots,\ \partial P_i/\partial \delta_n]$$

$$\Delta\delta = \mathrm{col}[\Delta\delta_1,\ \Delta\delta_2,\ \ldots,\ \Delta\delta_n]$$

$$H = [\partial P_i/\partial U_1,\ \partial P_i/\partial U_2,\ \ldots,\ \partial P_i/\partial U_n]$$

$$\Delta U = \mathrm{col}[\Delta U_1,\ \Delta U_2,\ \ldots,\ \Delta U_n]$$

when $i \neq j$

$$\partial P_i/\partial \delta_j = -\,|\boldsymbol{U}_i||U_j|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij})$$

$$\partial P_i/\partial \boldsymbol{U}_j = -\,|\boldsymbol{U}_i||U_j|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij})$$

when $i=j$,

$$\partial P_i/\partial \delta_i = |\boldsymbol{U}_i|\sum_{\substack{j=1 \\ i\neq j}}^{n}|U_j|(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij})$$

$$\partial P_i/\partial \boldsymbol{U}_i = -\,|\boldsymbol{U}_i|\sum_{\substack{j=1 \\ i\neq j}}^{n}|U_j|(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}) - 2\boldsymbol{U}_i^2 G_{ii}$$

If it is assumed that the voltage at each bus remains unchanged when only the active outputs of a few generators are changed, the third part in the right side of Equation (3.26) could be ignored. Therefore, Equation (3.26) can be simplified as:

$$P_i = P_i^0 + J\Delta\delta$$

Then, one will have:

$$\Delta P_i = J\Delta\delta$$

It can be written in the form of matrix:

$$\Delta P = \boldsymbol{J}\Delta\delta$$

where,

ΔP: the vector of node active power variations

$$\boldsymbol{J} = \begin{bmatrix} \partial P_1/\partial\delta_1, & \partial P_1/\partial\delta_2, & \ldots, & \partial P_1/\partial\delta_n \\ \partial P_2/\partial\delta_1, & \partial P_2/\partial\delta_2, & \ldots, & \partial P_2/\partial\delta_n \\ \ldots & \ldots & \ldots & \ldots \\ \partial P_n/\partial\delta_1, & \partial P_n/\partial\delta_2, & \ldots, & \partial P_n/\partial\delta_n \end{bmatrix}$$

Because $\boldsymbol{J}$ is a singular matrix, its inverse matrix $\boldsymbol{J}^1$ doesn't exist. If a bus, for example, a load Bus k is selected as a reference bus, and delete row k and column k

from J, the matrix J will become a non-singular matrix. Therefore, the increment of power angle, $\Delta\delta$ can be obtained by the following formula:

$$\Delta\delta = J^{-1}\Delta P$$

The formula used to calculate the increment of active power flow in transmission Line ij will be derived next. If I_{ij} is the current flowing through the Line ij, the power flow through the Line ij is:

$$
\begin{aligned}
S_{ij} &= P_{ij} + \mathrm{j}Q_{ij} = \boldsymbol{U}_i \boldsymbol{I}_{ij}^* \\
&= |\boldsymbol{U}_i|(G_{ij} - \mathrm{j}B_{ij})(|\boldsymbol{U}_i| - |\boldsymbol{U}_j|\mathrm{e}^{-\mathrm{j}(\delta_i - \delta_j)}) \\
&= |\boldsymbol{U}_i|(G_{ij} - \mathrm{j}B_{ij})\{|\boldsymbol{U}_i| - |\boldsymbol{U}_j|[\cos(\delta_i - \delta_j) - \mathrm{j}\sin(\delta_i - \delta_j)]\} \\
&= |\boldsymbol{U}_i|G_{ij}[|\boldsymbol{U}_i| - |\boldsymbol{U}_j|\cos(\delta_i - \delta_j)] - |\boldsymbol{U}_i|B_{ij}|\boldsymbol{U}_j|\sin(\delta_i - \delta_j) \\
&\quad + \mathrm{j}\{|\boldsymbol{U}_i|G_{ij}|\boldsymbol{U}_j|\sin(\delta_i - \delta_j) - |\boldsymbol{U}_i|B_{ij}[|\boldsymbol{U}_i| - |\boldsymbol{U}_j|\cos(\delta_i - \delta_j)]\} \\
&= G_{ij}[|\boldsymbol{U}_i|^2 - |\boldsymbol{U}_i||\boldsymbol{U}_j|\cos(\delta_i - \delta_j)] + B_{ij}|\boldsymbol{U}_i||\boldsymbol{U}_j|\sin(\delta_i - \delta_j) \\
&\quad + \mathrm{j}\{G_{ij}|\boldsymbol{U}_i||\boldsymbol{U}_j|\sin(\delta_i - \delta_j) - B_{ij}[|\boldsymbol{U}_i|^2 - |\boldsymbol{U}_i||\boldsymbol{U}_j|\cos(\delta_i - \delta_j)]\}
\end{aligned}
$$

The active power flow through transmission Line ij is:

$$P_{ij} = G_{ij}[|\boldsymbol{U}_i|^2 - |\boldsymbol{U}_i||\boldsymbol{U}_j|\cos(\delta_i - \delta_j)] + B_{ij}|\boldsymbol{U}_i||\boldsymbol{U}_j|\sin(\delta_i - \delta_j) \qquad (3.27)$$

If we assume $|\boldsymbol{U}_i| \approx |\boldsymbol{U}_j|$, and $\cos(\delta_i - \delta_j) \approx 1$, Equation (3.27) can be simplified as:

$$P_{ij} = B_{ij} * |\boldsymbol{U}_i||\boldsymbol{U}_j|\sin(\delta_i - \delta_j) \qquad (3.28)$$

Let P_{ij}^0 be the initial active power flow through Line ij and ΔP_{ij} be the increment of active power flow in Line ij caused by the redispatching of the active power output of generators, then, the following expression holds:

$$
\begin{aligned}
P_{ij} &= P_{ij}^0 + \Delta P_{ij} \\
&= G_{ij}[|\boldsymbol{U}_i^0|^2 - |\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 + \Delta\delta_i - \delta_j^0 - \Delta\delta_j)]
\end{aligned}
$$

$$+ B_{ij}*|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\sin(\delta_i^0 + \Delta\delta_i - \delta_j^0 - \Delta\delta_j)$$

$$= G_{ij}*[|\boldsymbol{U}_i^0|^2 - |\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)\cos(\Delta\delta_i - \Delta\delta_j) + |\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\sin(\delta_i^0 - \delta_j^0)\sin(\Delta\delta_i - \Delta\delta_j)]$$

$$+ B_{ij}*|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\sin(\delta_i^0 - \delta_j^0)\cos(\Delta\delta_i - \Delta\delta_j) + B_{ij}*|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)\sin(\Delta\delta_i - \Delta\delta_j)$$

$$\approx G_{ij}*[|\boldsymbol{U}_i^0|^2 - |\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)] + B_{ij}*|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\sin(\delta_i^0 - \delta_j^0)$$

$$+ B_{ij}*|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)\sin(\Delta\delta_i - \Delta\delta_j)$$

Obviously, the increment of active power flow in Line ij is:

$$\Delta P_{ij} = B_{ij}|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)\sin(\Delta\delta_i - \Delta\delta_j)$$

$$\approx [B_{ij}|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)] * (\Delta\delta_i - \Delta\delta_j) \qquad (3.29)$$

Let M denote the line-bus incidence matrix. Also, let Y be the diagonal matrix with diagonal element $Y_{kk} = B_{ij}|\boldsymbol{U}_i^0||\boldsymbol{U}_j^0|\cos(\delta_i^0 - \delta_j^0)$ (for example, there are n lines in the system, and we designate the line between Buses i and j as Line k) and let $\boldsymbol{\Delta F}$ be the vector of increments of real power flow through the lines. Then, we can write $\boldsymbol{\Delta F}$ in the form of a matrix as:

$$\begin{aligned}\boldsymbol{\Delta F} &= Y*M*\Delta\delta \\ &= Y*M*\boldsymbol{J}^{-1}\Delta P \\ &= S*\Delta P \qquad (3.30) \\ S &= Y*M*\boldsymbol{J}^{-1}\end{aligned}$$

Matrix S is the sensitivity matrix of power increments in transmission lines for power increment in each node.

In order to explain Equation (3.30), assume that there is a line k located between Buses i and j, and let $Y(k)$ denote the kth row of matrix Y and $M(k)$ indicates the kth row of matrix M.

$$\Delta F(k) = \Delta P_{ij} = Y(k) * M(k) * \Delta\delta$$

where,

$$Y(k) = [0, 0, \ldots, \underset{\text{the } k\text{th element}}{\underbrace{\{-B_{ij}|U_i^0||U_j^0|\cos(\delta_i^0 - \delta_j^0)\}}}, \ldots, 0]$$

$$M(k) = [0, 0, \ldots, \underset{\text{the } i\text{th element}}{\underline{-1}}, 0, \ldots\ldots 0, \underset{\text{the } j\text{th element}}{\underline{1}}, 0, \ldots, 0]$$

It is assumed that the positive direction of active power flow is from the node with a lower number to the node with a higher number. Also, assume that if active power is flowing into the Node *i*, its corresponding element in matrix *M* is 1, otherwise, it is –1. According to this principle, the line-bus incidence matrix *M* can be formed.

If the load demand is fixed, only the active outputs of related generators are changed to relieve the congestion, i.e.,

$$\Delta P = \Delta P_{\mathrm{G}}$$

Therefore, Equation (3.30) may be written as:

$$\begin{aligned} \Delta F &= Y^* M^* J^{-1} \Delta P \\ &= Y^* M^* J^{-1} \Delta P_{\mathrm{G}} \\ &= S^* \Delta P_{\mathrm{G}} \end{aligned}$$

In a system, its ISO will select a power plant to take the responsibility for compensating the transmission loss. Assume that this power plant is at Bus 1. Also assume that transmission Line k becomes congested when original generation schedule is executed. To relieve the congestion, the original generation schedule has to be changed. It means that some generators' outputs have to be reduced, and at the same time, others' outputs should be increased. One must determine which generators should increase their active power outputs, and which generators should decrease their active power outputs. Let ΔP_g be the reloading step size, ΔF_k be the power flow variation of Line k and $S_{k(i)}$ be the sensitivity coefficient of Generator i to ΔF_k. If there are m generation buses in the system then we can write:

$$\Delta F_k = S_{k(1)}*\Delta P_{G1} + \ldots + S_{k(i)}*\Delta P_{Gi} + \ldots + S_{k(j)}*\Delta P_{Gj} + \ldots + S_{k(m)}*\Delta P_{Gm}$$

If the output of the Generator at Bus i is increased by ΔP_g, and at the same time, the output of the generator at Bus j is decreased by ΔP_g, there will be a change of transmission loss, say ΔP_L. This loss increment has to be compensated by the generator at Bus 1. In this case, ΔP_L can be described as:

$$\begin{aligned}\Delta P_L = &(\partial P_L/\partial P_{G1})*\Delta P_{G1} +,\ldots,+ (\partial P_L/\partial P_{Gi})*\Delta P_{Gi}\\ &+,\ \ldots,+(\partial P_L/\partial P_{Gj})*\Delta P_{Gj}+,\ldots,+(\partial P_L/\partial P_{Gm})*\Delta P_{Gm}\\ &= (\partial P_L/\partial P_{G1})*\Delta P_L + (\partial P_L/\partial P_{Gi})*\Delta P_g - (\partial P_L/\partial P_{Gj})*\Delta P_g\end{aligned}$$

Therefore,

$$\Delta P_L = \frac{\partial P_L/\partial P_{Gi} - \partial P_L/\partial P_{Gj}}{1 - \partial P_L/\partial P_{G1}} * \Delta P_g$$

Equation (3.21) can be used to calculate $\partial P_L/\partial P_{Gi}$.

The power flow variation of Line k becomes:

$$\begin{aligned}\Delta F_k(i,j) &= S_{k(1)}*\Delta P_{G1} + \ldots +S_{k(i)}*\Delta P_{Gi} + \ldots+S_{k(j)}*\Delta P_{Gj} + \ldots+S_{k(m)}*\Delta P_{Gm}\\ &= S_{k(1)}*\Delta P_L + \ldots + S_{k(i)}*\Delta P_g + \ldots + S_{k(j)}*(-\Delta P_g) + \ldots + S_{k(m)}*0\\ &= S_{k(1)}*\Delta P_L + S_{k(i)}*\Delta P_g - S_{k(j)}*\Delta P_g\end{aligned}$$

The element $\Delta F_k(i,j)$ of the power flow variation matrix $\boldsymbol{\Delta F_k}$ represents the power flow variation due to an increase of ΔP_g MW at generation Bus i and a decrease of ΔP_g MW at generation Bus j. Replace the elements that will cause the congestion in Line k become more severe with zeros. If there are several transmission lines overloaded at the same time, one can form their corresponding power flow variation matrices respectively by keeping those elements which will not increase the flow of the already congested lines and letting others be replaced with zeros.

Next, form the corresponding incremental cost matrix for each power flow variation matrix. For example, if there exists a power flow variation matrix ΔF_k, i.e., $\Delta F_k(i,j)\neq 0$, its corresponding incremental cost $\Delta C_k(i,j)$ can be expressed in the following manner:

$$\Delta D_k(i,j) = C_1 * \Delta P_{G1} + C_i * \Delta P_{Gi} + C_j * (-\Delta P_{Gi})$$

$$= C_1 * \Delta P_L + C_i * \Delta P_g - C_j * \Delta P_g$$

$$\Delta C_k(i,j) = \frac{\Delta D_k(i,j)}{|\Delta F_k(i,j)|} \tag{3.31}$$

$\Delta D_k(i,j)$ is the variation of generation cost caused by an increase of 1 MW at generation Bus i and an decrease of 1 MW at generation Bus j. Element $\Delta C_k(i,j)$ means that if the flow in Line k is decreased by 1 MW by redispatching the active outputs at generation Bus i and j $(\Delta P_{Gi} + \Delta P_{Gj} = 0)$, the generation cost will increase by $\Delta C_k(i,j)$. From the incremental cost matrix ΔC_k, one can obtain the most economical way to relieve the congestion in Line k.

Assume that transmission Lines k and h are overloaded. The best way to relieve the congestion in transmission Line k is to increase the active power output at Bus i and decrease the output at Bus j. And the best way to relieve the congestion in transmission Line h is to increase the active power output at Bus p and decrease the output at Bus q. With these redispatches, the change in transmission loss becomes:

$$
\begin{aligned}
\Delta P_L &= (\partial P_L/\partial P_{G1})*\Delta P_{G1} +,\ldots,+ (\partial P_L/\partial P_{Gi})*\Delta P_{Gi} +,\ldots,+ (\partial P_L/\partial P_{Gj})*\Delta P_{Gj} \\
&\quad +,\ldots,+(\partial P_L/\partial P_{Gp})*\Delta P_{Gp} +,\ldots,+ (\partial P_L/\partial P_{Gq})*\Delta P_{Gq},+\ldots,+ (\partial P_L/\partial P_{Gm})*\Delta P_{Gm} \\
&= (\partial P_L/\partial P_{G1})*\Delta P_L + (\partial P_L/\partial P_{Gi})*\Delta P_g - (\partial P_L/\partial P_{Gj})*\Delta P_g \\
&\quad + (\partial P_L/\partial P_{Gp})*\Delta P_g - (\partial P_L/\partial P_{Gq})*\Delta P_g
\end{aligned}
$$

$$
\begin{aligned}
\Delta P_L &= \frac{\partial P_L/\partial P_{Gi} - \partial P_L/\partial P_{Gj}}{1 - \partial P_L/\partial P_{G1}}*\Delta P_g + \frac{\partial P_L/\partial P_{Gp} - \partial P_L/\partial P_{Gq}}{1 - \partial P_L/\partial P_{G1}}*\Delta P_g \\
&= \Delta P_{Lij} + \Delta P_{Lpq}
\end{aligned}
$$

ΔP_{Lij} is caused by the redispatching of generators at Buses i and j and ΔP_{Lij} is caused by the redispatching of generators at Buses p and q. Therefore:

$$
\Delta P_G = \begin{bmatrix} \Delta P_{Lij} \\ \cdots \\ \Delta P_{Gi} \\ \cdots \\ \Delta P_{Gj} \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta P_{Lpq} \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \cdots \\ \Delta P_{Gp} \\ \cdots \\ \Delta P_{Gq} \\ \cdots \\ 0 \end{bmatrix}
$$

$$= \begin{bmatrix} \Delta P_L \\ \dots \\ \Delta P_{Gi} \\ \dots \\ \Delta P_{Gj} \\ \dots \\ \Delta P_{Gm} \\ \dots \\ \Delta P_{Gn} \\ \dots \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta P_L \\ \dots \\ \Delta P_g \\ \dots \\ -\Delta P_g \\ \dots \\ \Delta P_g \\ \dots \\ -\Delta P_g \\ \dots \\ 0 \end{bmatrix}$$

$$\Delta F = Y^*M^*J^{-1}\Delta P$$

$$= Y^*M^*J^{-1}\Delta P_G$$

$$= S^*\Delta P_G$$

The line flows can be expressed in the form of the following sequence:

$$F^{(k+1)} = F^{(k)} + \Delta F;$$

There are two cases that should be considered. One is that power variation ΔP_G can't cause previously non-congested lines overloaded. If that happens, replace those corresponding elements in matrices ΔF_k and ΔC_k with zeros. And then, find a new most efficient way in the incremental cost matrix ΔC to replace the old one. Generally, the transmission system is designed to be robust. Therefore, this situation rarely occurs. Another case that it should be considered is that, if the power variation ΔP_G makes a congested line back to normal operating state, this transmission line will be treated as non-congested line, and only those transmission lines which are still overloaded are considered. It should be noted that, at the beginning, a transmission line returning to normal is actually in a state near the operating limit. Therefore, it should be observe its power flow change carefully. Once this line becomes overloaded again, the above-mentioned process should be used to treat it.

After each iteration, the active output of each generator should be checked. The output will be fixed if a generator reaches its generation limit. And then, find a new most economic way in the incremental cost matrix ΔC to replace the old one.

Therefore, the new power vector of the system becomes:

$$P^{(k+1)} = P^{(k)} + \Delta P_{G}$$

where, k is the number of iterations.

For the first iteration,

$$P^{1} = P^{0} + \Delta P_{G};$$

where, P^0 is the initial line flow vector under the original generation schedule.

Using $\Delta\delta = J^{-1}\Delta P = J^{-1}\Delta P_G$, the increment of power angle at each node can be obtained. The new power angle vector is:

$$\delta^{(k+1)} = \delta^{(k)} + \Delta\delta$$

For the first iteration,

$$\delta^{1} = \delta^{0} + \Delta\delta;$$

where, δ^0 is the initial power angle vector under the original generation schedule.

Using the new $P^{(k+1)}$ and $\delta^{(k+1)}$ to calculate the new $\partial P_l/\partial P_{Gi}$, matrix J and Y.

Iterative calculations will continue until the congestion disappears.

The system shown in Fig. 3.1 in Section 3.1.1 will be used to provide numerical examples. In this system, assume that all lines have the same resistance of 0.02 p.u. and reactance of 0.1 p.u. The voltages at Nodes 1, 2, 3 and 5 are all 1.05 p.u. The power factors of Loads 4 and 6 are 0.9. Voltage grade of the system is 138 KV. Base MVA is 100 MVA. Also, assume that there is a generator at Bus 3 to compensate the transmission loss. The price for loss compensation is $40/MWh. All other conditions are same as mentioned in Section 3.1.1.

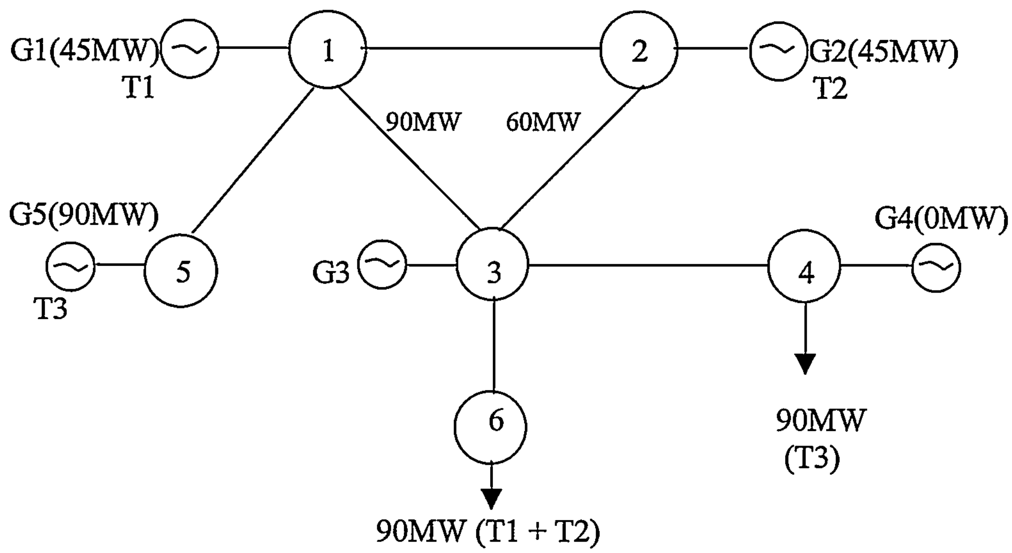


Fig. 3.4 A Deregulated System with a Generator as Loss Compensation Unit

First, the Newton-Raphson power flow analysis is used to obtain the initial power flow result based on the conditions described above. Tables 3.10 and 3.11 show the calculation results:

Table 3.10: Original Active Power Output of Each Generator (MW)

Generator No.	Active Power Output
G1	45
G2	45
G3	8.8
G4	0
G5	90

System Transmission Loss = 8.8 MW

Table 3.11: Original Active Power Flow in Each Line (MW)

Line No.	From	To	Power Flow
1	1	2	29.45
2	1	3	**104.05**
3	1	5	- 88.49
4	2	3	**74.29**
5	3	4	92.06
6	3	6	92.06

It can be noticed that Lines 2 and 4 are overloaded by 14.05 MW and 14.29 MW respectively. An optimal generation redispatch should be found by the ISO to relieve the congestions in Line 2 and Line 4. Table 3.12 shows the initial Power angle and voltage at each node for the original generation case.

Table 3.12: Initial Power Angle and Voltage at Each Node

Bus No.	Power Angle	Voltage
1	0.1761	1.05
2	0.1483	1.05
3	0.0787	1.05
4	0	0.98
5	0.2603	1.05
6	0	0.98

In order to begin the redispatch process, set the increment of generation, ΔP_g to 1 MW for each iteration. A redispatch schedule can be obtained by utilizing the proposed AC optimal power flow programming technique. The iteration process will stop when all the congestions are eliminated. For the given example, the optimal redispatch schedule is achieved after 15 iterations. Table 3.13 shows the redispatch for each iteration:

Table 3.13: Calculation Result of Generation Redispatch (MW)

Iteration No.	G1	G2	G3	G4	G5	Transmission Loss
0	45.0000	45.0000	8.8000	0	90.0000	8.8000
1	45.0000	44.0000	8.6217	2.0000	89.0000	8.6217
2	45.0000	43.0000	8.4435	4.0000	88.0000	8.4435
3	45.0000	42.0000	8.2655	6.0000	87.0000	8.2655
4	45.0000	41.0000	8.0876	8.0000	86.0000	8.0876
5	45.0000	40.0000	7.9099	10.0000	85.0000	7.9099
6	45.0000	39.0000	7.7323	12.0000	84.0000	7.7323
7	45.0000	38.0000	7.5548	14.0000	83.0000	7.5548
8	45.0000	37.0000	7.3775	16.0000	82.0000	7.3775
9	45.0000	36.0000	7.2003	18.0000	81.0000	7.2003
10	45.0000	35.0000	7.0232	20.0000	80.0000	7.0232
11	45.0000	34.0000	6.8463	22.0000	79.0000	6.8463
12	45.0000	33.0000	6.6695	24.0000	78.0000	6.6695
13	45.0000	32.0000	6.4928	26.0000	77.0000	6.4928
14	45.0000	31.0000	6.3162	28.0000	76.0000	6.3162
15	45.0000	30.0000	6.1397	30.0000	75.0000	6.1397

The final result of active power flow in each transmission line based on the new generation schedule is listed in Table 3.14:

Table 3.14: Final Active Power Flow in Each Line on the Base of New Generation Schedule (MW)

Line No.	From	To	Power Flow
1	1	2	29.6192
2	1	3	89.6034
3	1	5	-73.7199
4	2	3	59.6434
5	3	4	61.6684
6	3	6	91.5467

In order to show the accuracy of the line flows shown in Table 3.14, the power flow can be re-evaluated using the Newton-Raphson power flow analysis. The power flow results obtained using the N-R technique are shown in Table 3.15:

Table 3.15: Result of Power Flow Analysis (MW)

Line No.	From	To	Power Flow
1	1	2	29.6188
2	1	3	89.6034
3	1	5	-73.9523
4	2	3	59.4542
5	3	4	61.2968
6	3	6	92.0648

Transmission Loss = 6.4096 MW

Table 3.16 shows the errors between the power flows obtained by the proposed optimal programming technique and the Newton-Raphson power flow analyses method.

Table 3.16: Power Flow Error (MW)

Line No.	From	To	Power Flow Error
1	1	2	29.6188-29.6192=-0.0004
2	1	3	89.6034-89.6034=0.0000
3	1	5	-73.9523-(-73.7199)=-0.2324
4	2	3	59.4542-59.6434=-0.1892
5	3	4	61.2968-61.6684=-0.3716
6	3	6	92.0648-91.5467=0.5181

Transmission Loss Error = 6.4096 - 6.1397 = 0.2699 MW

Transmission Loss Error Rate = 0.2699/6.4096 = 4.21%

It can be concluded from the results in Table 3.16 that errors are very small. It indicates that the proposed AC optimal power flow programming technique can be used to obtain the generation redispatch scheme for relieving system congestion. Generation schedule obtained by the AC method is the same as that obtained by the DC method although the transmission loss is ignored in the DC method.

The total cost of the redispatch for congestion relief, based on the bid prices, is:

$$C_t = C_2 * \Delta P_{G2} + C_4 * \Delta P_{G4} + C_5 * \Delta P_{G5}$$

$$= -30*15 + 70*30 + (-40)*15$$

$$= \$1050/\text{hr}$$

If the original generation schedule is executed, the transmission loss is 8.8 MW. In terms of the new generation schedule, the transmission loss will drop to 6.4096 MW. It indicates that the new generation schedule will save:

$$8.8 - 6.4096 = 2.3904 \text{ MW}$$

$$2.3904 \text{ X } \$40 = \$95.616/\text{hr}$$

Therefore, the actual increased cost caused by the redispatch is:

$$\$1050 - \$95.616 = \$954.384/\text{hr}$$

Employing the calculation process mentioned above, the increased cost for a whole year could be easily obtained. If the increased cost due to redispatch is greater than the yearly cost of a new transmission line, then, building this new line will be a reasonable choice for the system operator and customers from the point of economic operation and system reliability. Adding a new transmission line into operation, in general, will cause the transmission loss of the whole system decline due to the redistribution of the power flows in transmission lines. It will obviously bring benefit to the users. For example, assume that a line is built between Nodes 3 and 5. Also assume that the reactance and resistance of this line is 0.11 p.u. and 0.022 p.u. respectively. Table 3.17 shows the power flows in the system with the new line under original generation schedule:

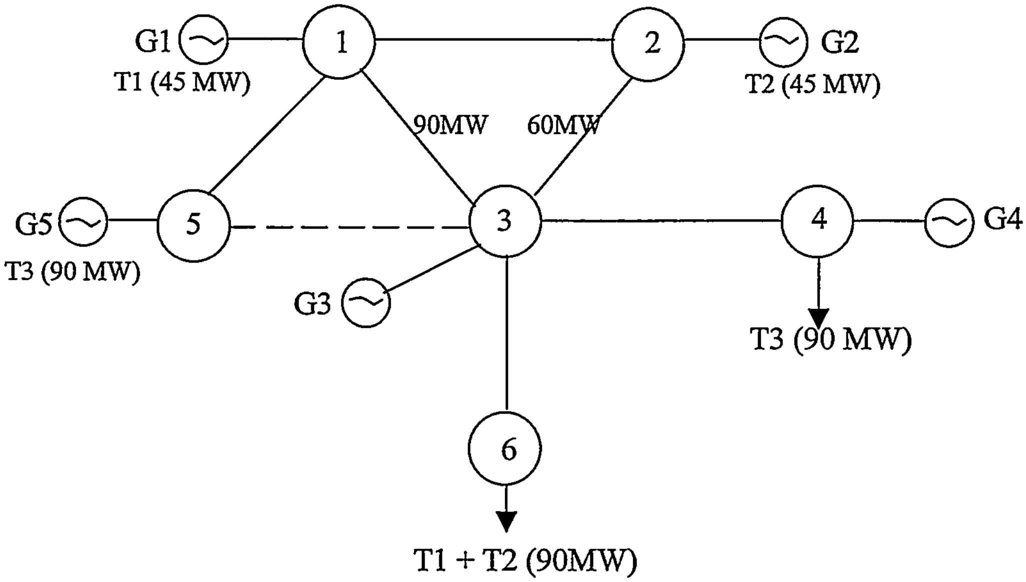


Fig. 3.5 A Deregulated System with Addition of a New Line

Table 3.17: Active Power Flow in Each Line with Addition of a New Line (MW)

Line No.	From	To	Power Flow	Operating Limit
1	1	2	6.45	150
2	1	3	**57.95**	**90**
3	1	5	- 19.40	150
4	2	3	**51.44**	**60**
5	3	4	92.06	150
6	3	6	92.06	150
7 (new)	3	5	- 69.51	150

System Transmission Loss = 6.3495 MW

Obviously, adding this new line to the system will relieve its congestion problem. At the same time, system transmission loss will also decrease from 8.8MW (without adding the new line) to 6.3495 MW (with the new line added) if the original generation schedule is executed. Therefore, the additional benefit brought by building the new transmission line is:

$40 X (8.8 - 6.3495) = $98.02/hr

3.3 Interruptible Loads in Generation Rescheduling

A power system in general serves two types of loads; named firm loads and interruptible loads. Firm loads must be supplied at all time except the occurrence of a system failure. Therefore, this kind of load is not considered by the ISO as a means to relieve system congestion. Interruptible loads can be redispatched by the ISO to relieve congestion. Interruptible load can be cut off at any time by the ISO as long as the loss to the customers is compensated through "lost opportunity payments". Customers with interruptible loads can bid for the compensating prices for the curtailment of their loads. The load curtailment will bring benefit to the customer if the bidding price for load cutback is higher than the price paid to the power supplier. A customer can make money from the difference between these two prices. For example, consider a customer with a certain amount of interruptible load has made a contract with a power supplier at a price of $50 for per MW per hour, and at the same time, this customer submitted a bid for load cutback at $100 per MW per hour to its ISO. If the ISO required him to reduce 20MW of its load from 9:00 to 12:00 am, this customer would make a profit of:

$$(\$100 - \$50) \times 20 \times 3 = \$3000$$

An interruptible load can be modeled as a fixed load plus a generator. This generator can be dispatched at any time by the ISO. For example, a customer needs 100 MW at normal state. This 100 MW load includes 40 MW interruptible load. Thus, this customer is equivalent to 100 MW of fixed load plus a 40 MW generator. The load model of this customer is shown in Fig. 3.6.

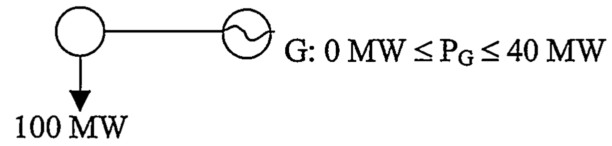


Fig. 3.6 Interruptible Load Model

With the help of the model shown in Fig. 3.6, an interruptible load can be considered by the ISO as a means to relieve congestion. The proposed optimal power flow method can be utilized to consider redispatch with interruptible loads.

In many cases, redispatching of generators alone can't solve the congestion problem. For example, in the system shown in Fig. 3.7, there are three transactions T1, T2 and T3. Transaction T1 requires the system to transmit 50 MW from point 1 to point 2. T2 and T3 require 40 MW and 50 MW transmitted in the same direction respectively.

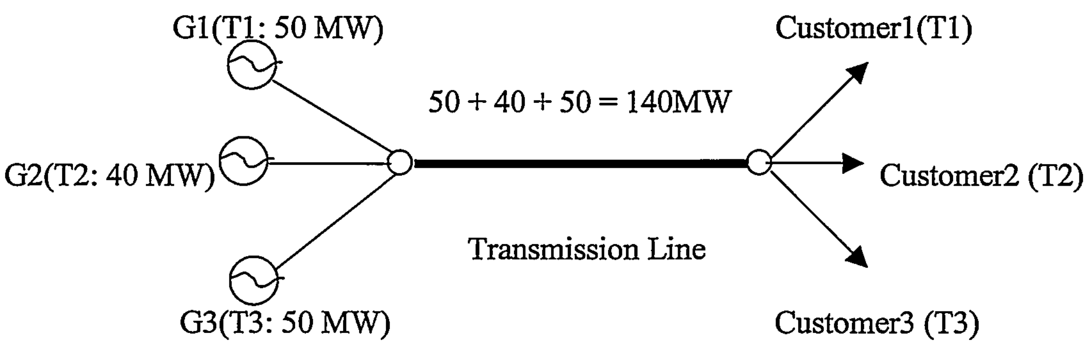


Fig. 3.7 A Deregulated System with an Interruptible Load

If the operating limit of the transmission line is 120 MW, congestion will occur with an overload of 20 MW. In the case that all three transactions are firm loads, an additional transmission line will be necessary to relieve the congestion. Assume that the investment cost of this new line is \$900,000/yr. If Customer 3 has 30 MW of interruptible load, the ISO can order it to cut back 20 MW to relieve the congestion. Assume that the power supply price for T3 is \$50/MWh, and Customer 3 bids

$80/MWh to the ISO for load curtailment. Also assume that G1, G2 and G3 would like to reduce their outputs at the payment prices of $40/MWh, $30/MWh and $20/MWh respectively. Therefore, G1 will be selected to decrease 20 MW output according to the optimal principle. The total congestion fee for one hour is:

$$20 \text{ MW} * \$80/\text{MWh} - 20 \text{ MW} * \$40/\text{MWh} = \$800/\text{hr}$$

If the congestion occurs during 10:00 am – 12:00 a.m. each day, the total congestion fee for one day is:

$$\$800 \text{ X } 2 = \$1{,}600/\text{per day}$$

The total congestion fee for one year is:

$$\$1{,}600 \text{ X } 365 = \$584{,}000/\text{yr}$$

Customer 3 will make a profit of:

$$(\$80 - \$50)*20\text{MW}*2*365 = \$438{,}000/\text{yr}$$

The congestion fee allocated to Customer 3 is:

$$\$584{,}000/\text{yr X} \frac{50 \text{ MW}}{50\text{MW} + 40\text{MW} + 50\text{MW}} = \$208{,}571/\text{yr}$$

The actual profit made by Customer 3 is:

$$\$438{,}000/\text{yr} - \$208{,}571/\text{yr} = \$229{,}429/\text{yr}$$

Due to its ability to interrupt Customer 3's load, the ISO will be able to relieve the congestion without having to build a new transmission line and, therefore, will save:

$$\$900{,}000/\text{yr} - \$584{,}000/\text{yr} = \$316{,}000/\text{yr}$$

The numbers suggest that curtailing the load of Customer 3 will be more economical than building a new transmission line.

Next, the system described in DC method in Section 3.1.1 will be utilized to illustrate the effect of interruptible load in making decisions with respect to transmission expansion. Assume Node 4 to be pure P-Q bus with an interruptible capacity of 40 MW. The price for cutting back 1 MW at Node 4 is $70/hr. As mentioned before, although Node 4 is a pure P-Q bus, it can be modelled as a fixed load of 90 MW plus a generator with a generation capacity of 40 MW. It means that for the purpose of analysis, Node 4 will be treated as a generation bus P-V bus. To avoid congestion, the user at Node 4 should cut back 30 MW of its load, and at the same time, both generators at Nodes 2 and 5 should decrease their outputs by 15 MW. With the help of the AC method, power flow in each line can be obtained for the new schedule by considering Node 4 as P-V bus and then as a P-Q bus. In the first case, assume that Node 4 has a load of 90 MW and a generator with capacity of 40 MW. Also assume that the voltages at Nodes 1, 2, 3 and 5 have the same magnitude of 1.05 p.u. The voltage at Node 4 is 1.0 p.u. The resulting power angles and line flows are shown in Tables 3.18, 3.19 and 3.20.

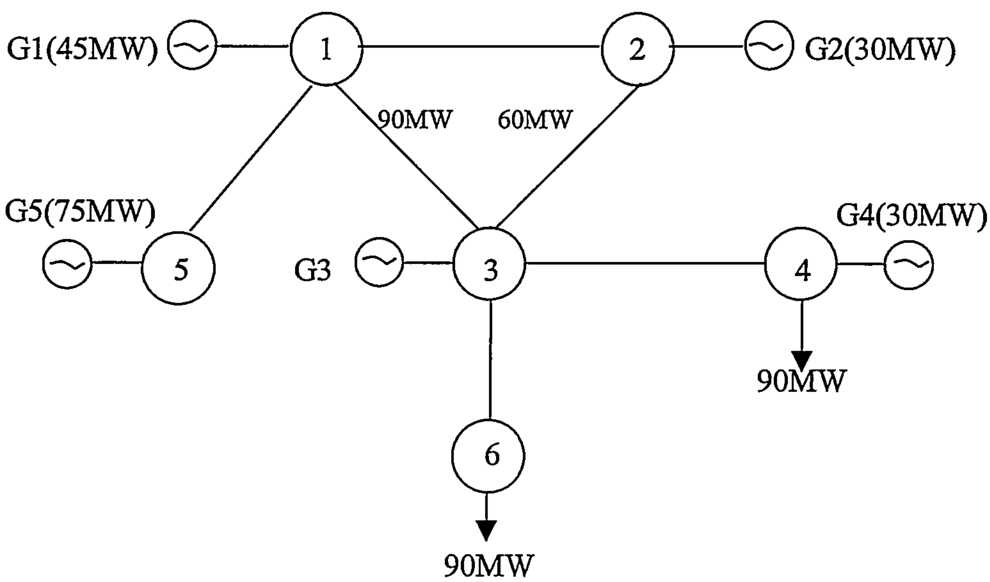


Fig. 3.8 A System with Interruptible Load Bus Considered as P-V Bus

Table 3.18: Power Angle and Voltage of Each Node with Node 4 Considered as P-V Bus

Bus No.	Power Angle	Voltage
1	0.1624	1.0500
2	0.1345	1.0500
3	0.0787	1.0500
4	0.0285	1.0000
5	0.2327	1.0500
6	0	0.9842

Table 3.19: Net Active and Reactive Power Injected into Each Node with Node 4 Considered as P-V Bus (P:MW, Q:MVar)

Bus No.	Active Power	Active Power
1	45.0000	-1.9912
2	30.0000	-3.8559
3	6.4096	130.4364
4	-60.0000	-36.6784
5	75.0000	-12.2759
6	-90.0000	-43.5870

Table 3.20: Power Flow in Each Line with Node 4 Considered as P-V Bus (P:MW, Q:MVar)

Line No.	From	To	Active Power Flow	Reactive Power Flow
1	1	2	29.6188	-5.4957
2	1	3	89.3334	-14.0101
3	1	5	-73.9523	17.5146
4	2	3	59.4542	-10.1748
5	3	4	60.9891	41.6237
6	3	6	92.0648	53.9111

System Transmission Loss = 6.4096 MW

In the second case, assume that the customer at Node 4 cut back 30 MW load, and at the same time, both generators at Nodes 2 and 5 decrease their corresponding outputs by 15 MW. Also assume that the voltages at Nodes 1, 2, 3 and 5 have the same magnitude of 1.05 p.u. Node 4 is now a pure P-Q bus. Tables 3.21, 3.22 and 3.23 show the bus angles and line flows when Node 4 is considered as a P-Q bus.

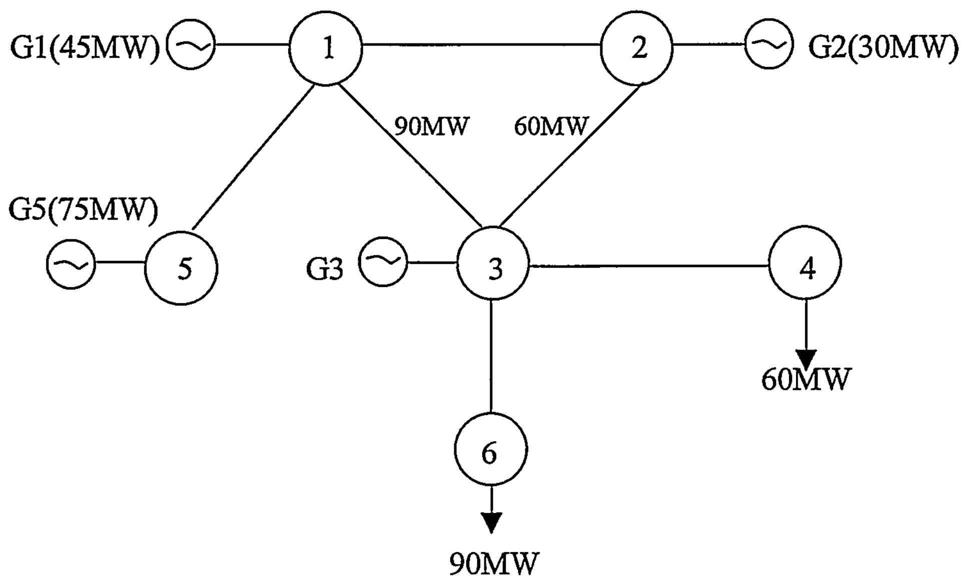


Fig. 3.9 A System with Interruptible Load Bus Considered as P-Q Bus

Table 3.21: Power Angle and Voltage of Each Node with Node 4 Considered as P-Q Bus

Bus No.	Power Angle	Voltage
1	0.1624	1.0500
2	0.1345	1.0500
3	0.0787	1.0500
4	0.0275	1.0079
5	0.2327	1.0500
6	0	0.9842

Table 3.22: Net Active and Reactive Power Injected into Each Node with Node 4 Considered as P-Q Bus (P:MW, Q:MVar)

Bus No.	Active Power	Active Power
1	45.0000	-1.9912
2	30.0000	-3.8559
3	6.2955	122.2457
4	-60.0000	-29.0580
5	75.0000	-12.2759.
6	-90.0000	-43.5870

Table 3.23: Power Flow in Each Line with Node 4 Considered as P-Q Bus (P:MW, Q:MVar)

Line No.	From	To	Active Power Flow	Reactive Power Flow
1	1	2	29.6188	-5.4957
2	1	3	89.3334	-14.0101
3	1	5	-73.9523	17.5146
4	2	3	59.4542	-10.1748
5	3	4	60.8750	33.4331
6	3	6	92.0648	53.9111

System Transmission Loss = 6.2955 MW

From the bus angles and line flows shown in Tables 3.18, 3.19, 3.20, 3.21, 3.22, and 3.23, it can be noticed that the results of power flow analysis with Node 4 modeled as a fixed load plus a generator is very close to that with considering Node 4 as a pure P-Q bus. Node 6 is selected as the reference bus in these two cases, power angles at Nodes 1, 2, 3 and 5 remain unchanged. Only the power angle at node 4 is changed. In the first case, the power angle at Node 4 is 0.0285 radian. In the second case, it is 0.0275 radian. The power angle difference is:

$$0.0285 - 0.0275 = 0.0010 \text{ (rad)}$$

The voltage difference at Node 4 between these two cases is:

$$|1.0000 - 1.0079| = 0.0079 \text{ (p.u.)}$$

The reason for this difference in voltage magnitude is that, in the first case the magnitude of voltage at Node 4 was held constant at 1.00 p.u. as it was considered as a generation bus. But in the second case, the magnitude of voltage at Node 4 was not a constant rather than it was determined by the operating conditions of the whole system.

Comparing the line flows shown in Table 3.20 and 3.23, it can be noticed that only the power flow in transmission Line 5 (between node 3 and node 4) has been changed when Node 4 was considered as a P-Q bus instead of a P-V bus. In the P-V bus case, the active and reactive power flows in Line 5 is 60.9891 MW and 41.6237 MVar respectively and, in the P-Q bus case, they are 60.8750 MW and 33.4331 Mvar respectively.

The difference in active power flows between these two cases is 0.1141 MW and the difference in reactive power flows between these two cases is 8.1906 Mvar. It shows that considering Node 4 as a P-V bus or a P-Q bus has little impact on the distribution of active power flows. But it has a considerable influence on the distribution of reactive power flows.

In addition, the difference between the transmission losses in both cases is very small. The difference is 1.78% which is mainly caused by the different distribution of reactive power flows in the two cases.

Analysis results show that modeling interruptible load as a fixed load plus a generator is practical and feasible when one calculates the redispatch of generation for a congested system. The advantage of this modeling is that treating the original variable load as fixed load will simplify the calculation process of determining generation reschedule without a significant loss of accuracy.

CHAPTER 4: INFLUENCE OF SHORT-TERM POWER MARKET TO TRANSMISSION SYSTEM EXPANSION PLANNING

4.1 Introduction to Power Market

Under deregulation, electricity producers compete with each other to sell electricity in an open market. In some systems, customers bid for the purchase of electricity. In others, customers simply notify the amount of their need to the system operators. The trade of electricity takes various forms under deregulation. Presently, electricity is traded in three different ways: Poolco, bilateral trading and power exchange market.

There is only one buyer in a Poolco system. A Poolco is a government or a quasi-governmental agency that buys for everyone. In a pure Poolco system, for example, ISO-New England and the England-Wales systems, users only submit the required energy amount to Poolco. Users don't need to submit purchasing price for the power. The job of the system operator is to run the system to assure the best reliability and lowest average cost for users. The system operator will endeavour to satisfy all users by purchasing power as cheaply as possible and averaging the cost over all users so that every user gets equal benefit of this optimized buying. But some Poolcos, such as Alberta Poolco and New York Poolco, require users to submit purchasing price and the power demand at the same time. Actually, this kind of Poolco combines ISO with power exchange market together. In this type of Poolco system, the system operator matches users and sellers who submit bids using a "non-discriminate auction"----a "stock market" type of bidding process where everyone transacts business at any one moment at the same price (Market Clearing Price), and everyone knows that price because it is posted. Although Alberta and New York system adopted the Poolco model to run power market and operate their system, they still encourage bilateral transactions. A bilateral transaction means individual power producers and users can make a deal to exchange power privately with contracted quantities and at contracted prices.

In California, the system operation is completely decoupled from the generation market. A completely independent system operator (ISO) operates the power grid throughout the state as a common carrier for all wholesale users. A non-profit power exchange corporation --- the California Power Exchange (CalPX), was set up in 1998 to handle the power spot market in a manner that is blind to system operation. The ISO and CalPX are independent entities, each functioning in an unbiased manner, i.e., no company or user is given any advantage over any others. And at the same time, long-term bilateral transactions are not encouraged in the California system according to the deregulation law passed in 1996, which wanted to create a full competitive market with transparent prices. On Jan. 23, 2001, CalPX went out of business due to some flaws in the deregulation plan such as mandatory Buy-Sell requirement, etc. Some changes have already been made to California power exchange market to address these problems, for example, cancelling the mandatory Buy-Sell requirement and encouraging long-term contracts. Presently, the California ISO is taking over the responsibility to run the spot market instead of the CalPX. Actually, present California model is very similar to the

Alberta model in managing power exchange market and rescheduling generation to relieve congestion.

Next, the Alberta model is used to illustrate some issues related to power exchange market. Generally, in Alberta, electric customers purchase electricity from power producers through two ways----bilateral contract and short-term power exchange market.

Power Pool of Alberta (also called Alberta ISO) operates a day-ahead short-term market consisting of 24-hour periods for the next day. Every morning, buyers submit their requirements for electric power for the next day, as 24-hour increments, along with their bid prices for purchase. Similarly, power producers also submit their bids (24-hour amounts and prices) to Pool. All bid prices submitted by the power producers or purchasers can't be changed for the next trading day. The Power Pool of Alberta matches buyers and sellers in a non-discriminate auction method on an hourly basis. This results in a forecasted market-clearing price that, like the stock market, fluctuates over time (every hour). Price may be high during peak times or during situations where some cheap supply is unavailable due to generation plant outages, etc. This bid information also allows the Power Pool to make forecasts of electricity supply and demand for the next day and publish a schedule for the next trading day.

As the demand for electricity shifts throughout the day, a real-time spot market is also established and used by Power Pool of Alberta to dispatch electricity to meet real-time load demand. The actual hourly market-clearing price is determined by real-time spot market in terms of the least cost principle. In short-term spot market, successful sellers are those who bid power at or below the market-clearing price.

Successful buyers are those who bid to buy at or above that price. All successful sellers in each hour period are paid the market-clearing price for that hour. Similarly, all successful buyers also pay the same price. From the view of ISO, this kind of power trade can be considered as a short-term multipoint to multipoint bilateral transaction. The simple system shown in Fig. 4.1 can be used to explain the determination of market clearing price.

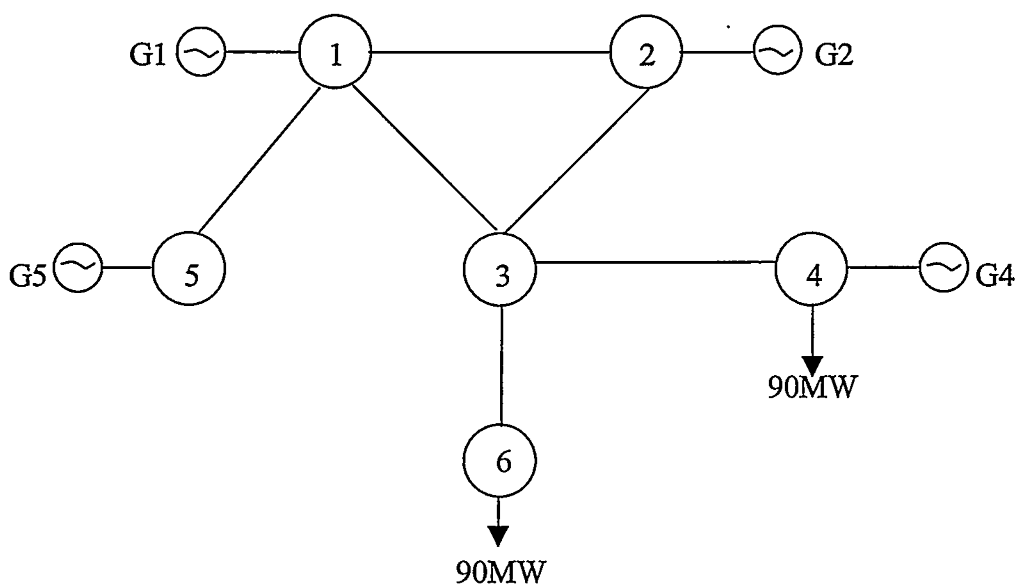


Fig. 4.1 A System Used to Illustrate the Market Clearing Price

There are four generators and two customers in the system as shown in Fig. 4.1. The generators are competing with each other in the short-term energy market to sell the electricity to the users. Meanwhile, the customers are also bidding their purchase prices for their loads. Fig. 4.2 represents the bidding prices submitted by power producers and buyers at a certain hour.

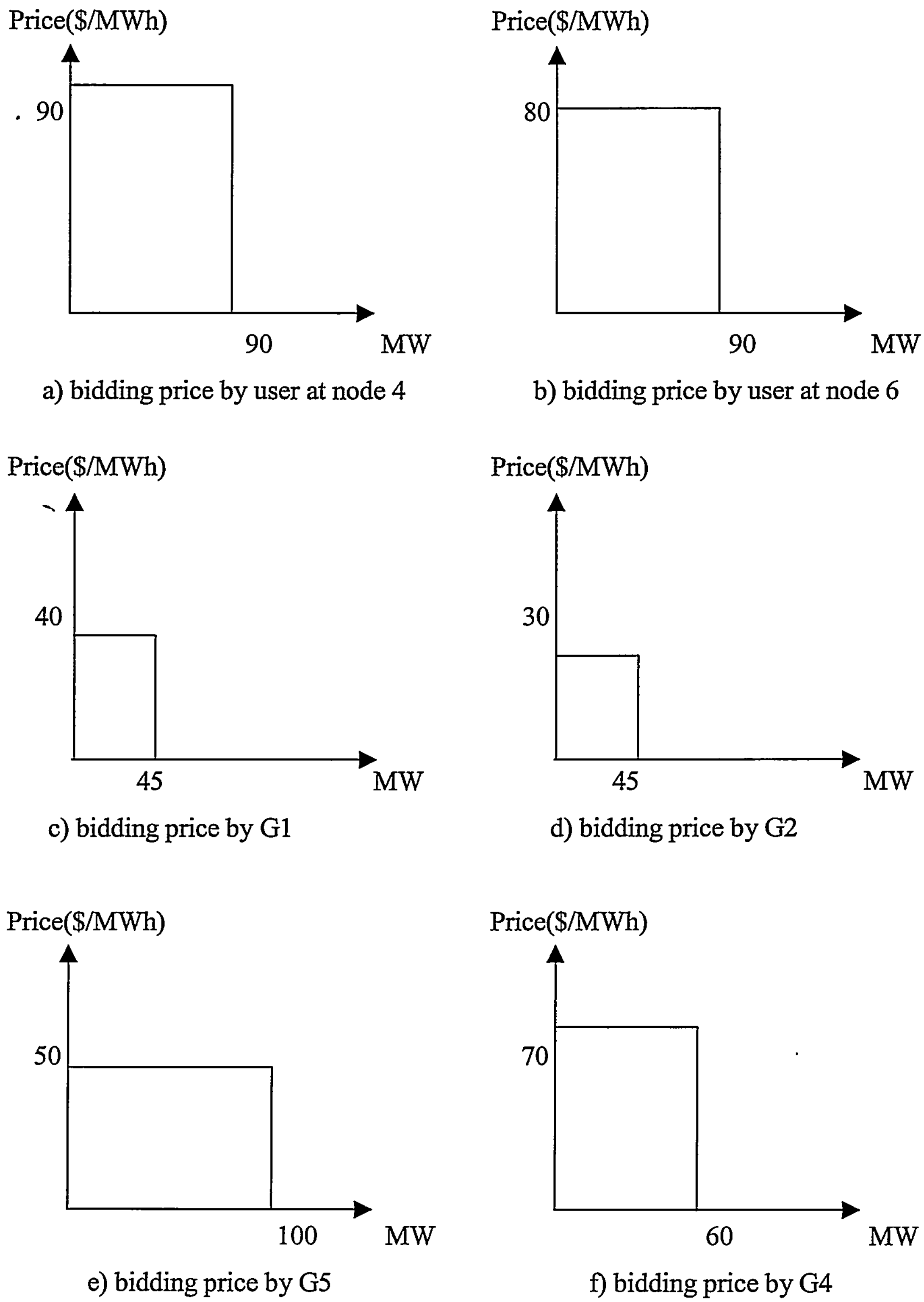


Fig. 4.2 Bidding Prices

The market-clearing price will be determined where the customers' bidding curve intersects with the generators' bidding curve as shown in Fig. 4.3.

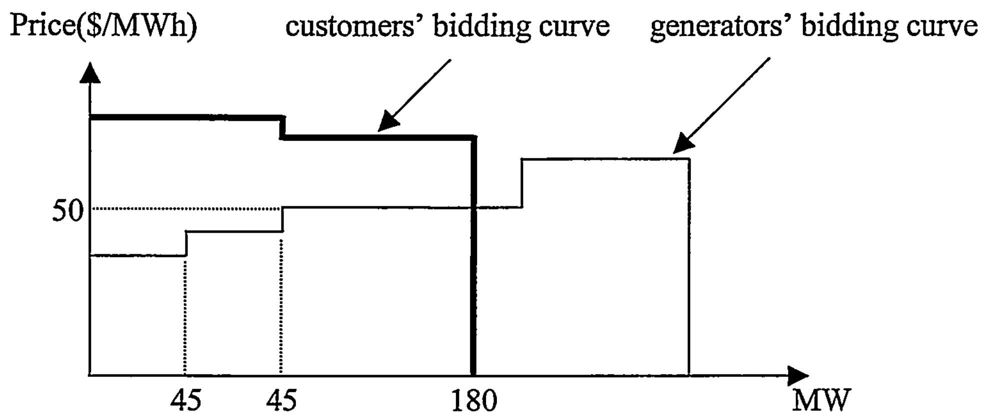


Fig. 4.3 Determination of Market Clearing Price

The market-clearing price shown in Fig. 4.3 is $50/MWh.

Besides the electricity trading in short-term market in Alberta, long-term private bilateral contracts are not only allowed, but also envisioned as the main means through which the majority of power will be bought and sold. It is understandable because the electricity buyers can obtain power at lower prices from the sellers if they could make a private deal with power sellers through long-term bilateral contracts. Similarly, the power producers also like this form of power exchange, because they can use this way to avoid the risk of price volatility and make a stable profit. In addition, power producers can efficiently schedule their generators' commitments in terms of the long-term bilateral contracts to increase the efficiency of their generators. But why Power Exchange Market (Spot Market) is still needed if the bilateral trading is considered as the most welcomed way to trade electricity? The reason is very simple: no user can make an accurate load forecast for next months or years. But they can make a load forecast with satisfactory accuracy for next day.

In order to minimize the electricity cost, users usually decompose their loads into two parts: base load (or called "must run" load) and variable load. These two parts of load are shown in Fig. 4.4:

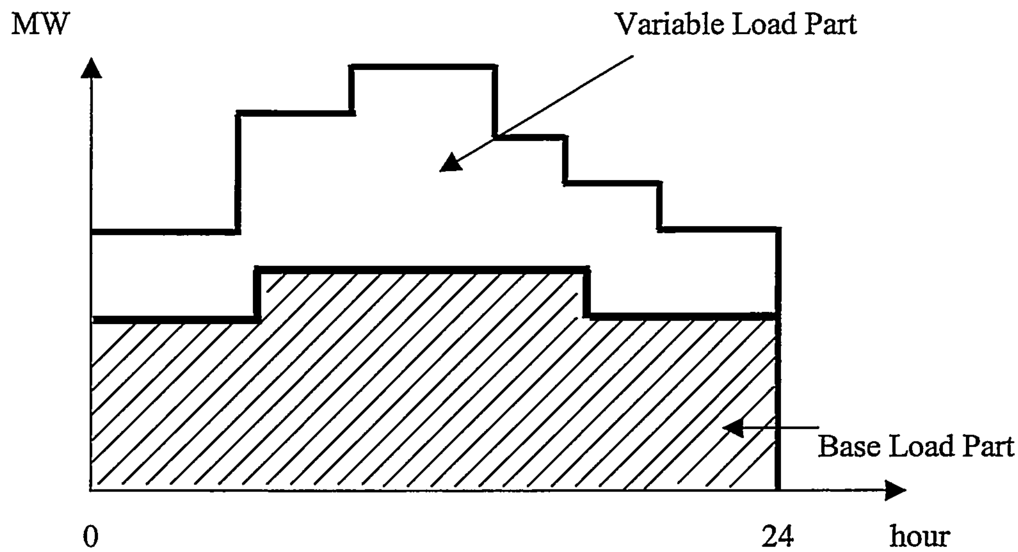


Fig. 4.4 Decomposed Load Components

Customers can predict their base loads easily through analyzing their previous load data. The base load is a long-term stable demand, which is not influenced by weather, temperature, holidays and economic circumstance, etc. Customers can purchase electricity for this part of load at a lower price through long-term bilateral trading with power producers.

Customers have to buy variable load in the Power Exchange Market (Short-term Market) to make up the difference between actual demand and base load. Because this part of load is influenced by external factors such as temperature, weather, etc., it is better to be forecasted a day ahead and traded through the spot market.

As soon as the Power Exchange Market matches up buyers and sellers to some equal level of *power sold* = *power bought*, the short-term generation schedule will be determined. The power companies will be required to submit the commitment of each generator to the ISO. And at the same time, the sellers and buyers must also submit their redispatch prices for each generator and interruptible load to the ISO. Redispatch prices show the seller's and buyer's willingness to vary their schedule up and down, at the request of the ISO to reduce the system congestion.

According to the rules and regulations, any long-term private bilateral contract made by buyers and sellers must be submitted to the ISO. The ISO uses this information to determine base load and base generation pattern for the whole system. Because majority of electricity is traded through bilateral transactions, the base load and base generation pattern are very important in determining system expansion plan. But only the long-term bilateral contracts data is not enough to determine whether a new transmission line is needed or not from the point of economy. The influence of short-term market should also be taken into consideration for system expansion plan in a deregulated system.

4.2 Short-Term Load Analysis

Short-term load forecasting is an important aspect of electricity trade in a deregulated system. A Poolco or an ISO makes important decisions with respect to spot market energy trade based on the short-term load forecast data provided by consumers and retailers. Each customer including electricity retailers will have to make a day ahead short-term load forecast for their variable load. In order to avoid penalty due to either over-estimating or underestimating its load, a customer has to make an accurate load forecast for the next day. A customer makes load forecast based on historical data, weather information, social and sporting events etc. with the help of a suitable mathematical/numerical program.

If some conditions, such as temperature, economic environment, etc., are unchanged, the variable load of any customer can be considered to be made up of three parts: constant part Q_0, time variable part $Q(t)$ and price responsive part $Q(r)$; where, r represents the market clearing price. It can be expressed as:

$$P(t) = Q_0 + Q(t) + Q(r); \quad t = 1, 2, 3, \ldots\ldots, 24$$

Q_0 and $Q(t)$ are the non-price responsive parts of a load. No matter how high the market clearing price is, Q_0 and $Q(t)$ would be dictated by the nature of the load and the time of the day. As we know, in terms of conventional classification, system load is generally classified as residential load, commercial load and industrial load. Among them, residential and commercial loads are almost non-price responsive. The reason is very simple, electricity interruption will bring great inconvenience to people's daily life and business activities. Therefore, no matter how high the price is, this kind of load must be guaranteed. The electricity retailers (they are also called customers by an ISO and generation companies) will bid much higher price than the market-clearing price for this kind of loads to ensure they could be supplied without being influenced by the fluctuation of spot price. But this kind of load is related to time, high in daytime and evening, low in midnight. Therefore, residential and commercial load can be considered to consist of Q_0 and $Q(t)$ only. Although residential and commercial loads change with a change in season, these loads fluctuate very slightly within a period with stable weather condition. In making a plan for system expansion, load variation due to seasonal changes should be taken into consideration. Most industrial loads are different from residential and commercial loads. Generally, industrial loads are insensitive to weather condition. But they are sensitive to electricity price. Some industrial users have interruptible loads to avoid using electricity when prices are high. For industrial users, their loads consist of three parts: Q_0, $Q(t)$ and $Q(r)$.

Under deregulation, load function $P(t)$ is a private information. No customer would be willing to provide this information to others including the ISO. Although customers' bids for electricity are blind to each other in the spot market (customers only know the market clearing price, they don't know others' bid prices), these bids are open to the ISO. As an electric system planner, the ISO could find out these three parts of $P(t)$ under certain conditions for each customer according to the historical bidding information submitted by each customer by employing an appropriate load analysis technique. Usually, different loads have different annual incremental rates. A system planner can easily obtain the average annual incremental rate by analyzing the historical load data, and thus, can acquire the typical daily load of each customer by using the following expression:

$$P_n(t) = P_0(t) + K^*P_0(t);$$

where, $P_n(t)$ represents the typical daily load in the future nth year. $P_0(t)$ is the initial daily load obtained through the historical load data of a customer. K is the average annual rate by which the customer's load increases.

4.3 Generator Bid Price Analysis

Once the typical daily load curve of each user is determined, system planners need to find out the economic benefit function of each generator, $B(P_i)$, where, P_i is the active output of Generator i. A generator's economic benefit function $B(P_i)$ is defined as generator's cost function $F(P_i)$ plus profit function $M(P_i)$. This relationship can be expressed as:

$$B(P_i) = F(P_i) + M(P_i)$$

Cost function $F(P_i)$ is a well-known quadratic function which can be described as:

$$F(P_i) = \alpha P_i^2 + \beta P_i + \Upsilon$$

Incremental cost function of Generator i, $C(P_i)$, is:

$$C(P_i) = 2\alpha P_i + \beta$$

The profit function $M(P_i)$ is influenced by the electrical market situation and power producer's sale strategy. The simplest profit function $M(P_i)$ is a linear function with active output P_i as independent variable. It can be expressed as:

$$M(P_i) = \mathrm{k}^* P_i;$$

where, k is the profit earned by per MWh of energy sale.

If $M(P_i)$ is a linear function, economic benefit function $B(P_i)$ can be written as:

$$\begin{aligned} B(P_i) &= F(P_i) + M(P_i) \\ &= \alpha P_i^2 + \beta P_i + \Upsilon + \mathrm{k}^* P_i \\ &= \alpha P_i^2 + (\beta+\mathrm{k}) P_i + \Upsilon \end{aligned}$$

Therefore, the bidding price function of Generator i, $H(P_i)$, can be obtained by differentiating the economic benefit function $B(P_i)$:

$$\begin{aligned} H(P_i) &= \partial B(P_i)/\partial P_i \\ &= 2\alpha P_i + (\beta+\mathrm{k}) \\ &= (2\alpha P_i + \beta) + \mathrm{k} \\ &= C(P_i) + \mathrm{k} \end{aligned}$$

For example, if the cost function of Generator i is:

$$F(P_i) = 0.024P^2 + 15P + 70$$

And if the profit function $M(P_i)$ of Generator i is:

$$M(P_i) = 10\,P_i$$

Then, the economic benefit function $B(P_i)$ is:

$$\begin{aligned} B(P_i) &= F(P_i) + M(P_i) \\ &= 0.024P_i^2 + 15P_i + 70 + 10{*}\,P_i \\ &= 0.024P_i^2 + 25P_i + 70 \end{aligned}$$

Therefore, the bidding price function of Generator i is:

$$\begin{aligned} H(P_i) &= 2{*}0.024{*}\,P_i + 25 \\ &= 0.048\,P_i + 25 \end{aligned}$$

The incremental cost function $C(P_i)$ of generator i is:

$$\begin{aligned} C(P_i) &= 2{*}0.024P_i + 15 \\ &= 0.048P_i + 15 \end{aligned}$$

Economic benefit function $B(P_i)$ and generation cost function $F(P_i)$ and profit function $M(P_i)$ of Generator i are shown in Fig. 4.5, incremental cost function $C(P_i)$ and bidding price function $H(P_i)$ of Generator i are shown in Fig. 4.6.

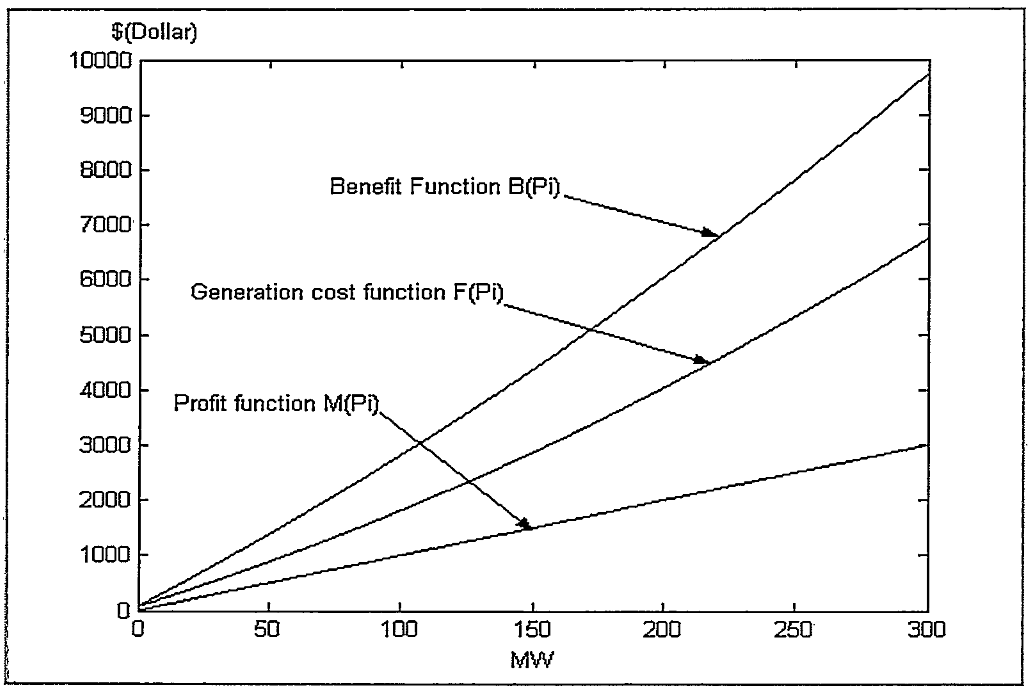


Fig. 4.5 Benefit Function, Generation Cost Function & Profit Function

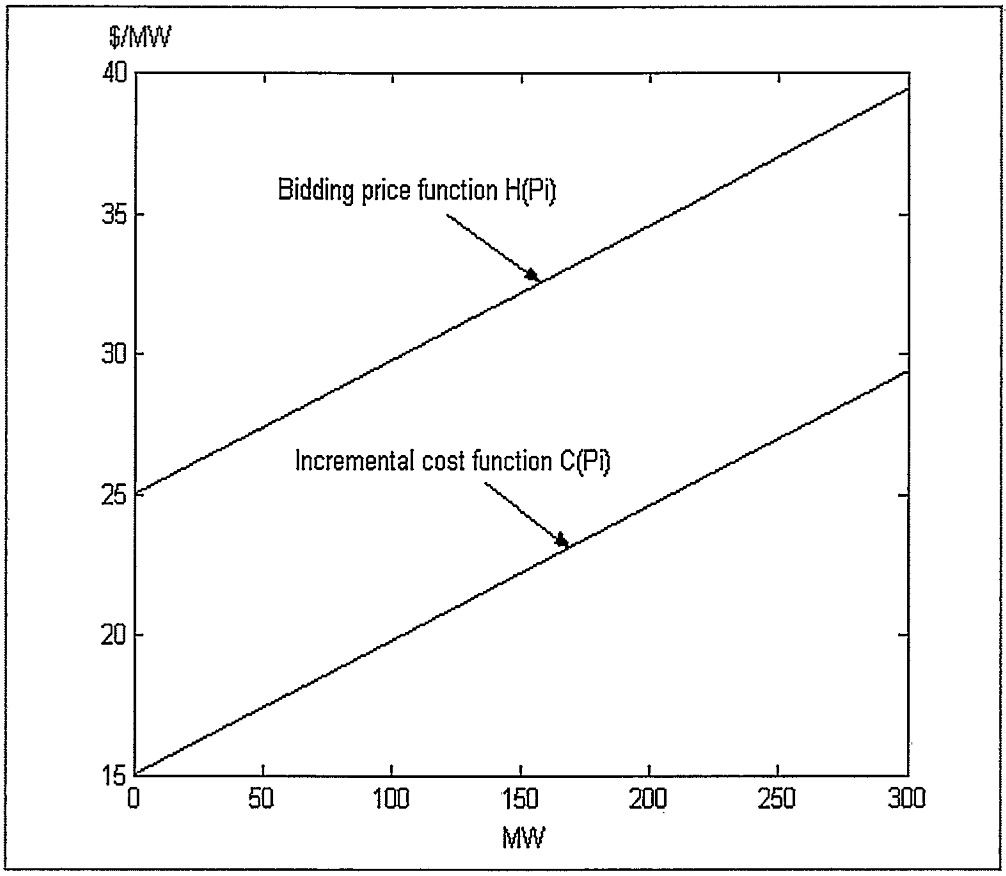


Fig. 4.6 Incremental Cost Function and Bidding Price Function

4.4 Short Term Generation Schedule Forecasting

An ISO considers economic benefit function $B(P_i)$ as the generation cost function of Generator i, and bidding price function $H(P_i)$ as the incremental cost function of generator i. Therefore, the original economic dispatch theory under regulated environment can be used to determine the generation schedule for the generators taking part in a short-term spot market. If the total load at a certain hour in a day is P_R, and the active output of Generator i which takes part in the spot market is P_i, then, the total generation cost from the point of the ISO is:

$$B_t = \sum_{i=1}^{n} B_i(P_i)$$

where,

B_i the money paid to generator i.

P_i load commitment of generator i.

N the number of generators taking part in the market.

One can minimize B_t for a given load P_R, with the following constraint:

$$\sum_{i=1}^{n} P_i - P_R = 0$$

In order to minimize B_t, a new non-constraint function B can be formed with the help of Lagrangian Multiplier λ.

$$B = B_t - \lambda \left(\sum_{i=1}^{n} P_i - P_R\right)$$

Minimum generation cost as viewed by the ISO for a given load is obtained when

$$\partial B / \partial P_i = \partial B_t / \partial P_i - \lambda \partial(\sum P_i - P_R) / \partial P_i = 0$$

$$\partial B_t / \partial P_i - \lambda(1-0) = 0$$

$$\partial B_t / \partial P_i = \lambda$$

Because

$$\partial B_t / \partial P_i = \partial(\sum B_i) / \partial P_i$$

$$= \partial B_i / \partial P_i$$

One can get:

$$\partial B_i / \partial P_i = \lambda$$

λ is called the system incremental cost. Actually, λ is the market-clearing price if the power producer is required to submit a linear bidding price function to the ISO. Assume that two generators are competing to sell power, and the load demand is P_R. The bidding price functions of Generator 1 and Generator 2 are shown in Fig. 4.7:

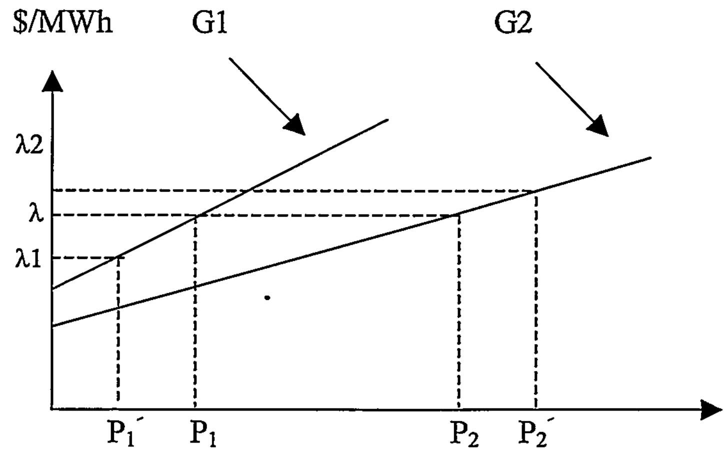


Fig 4.7 Unit Commitment Determination in terms of Generator's Bidding Price Function

According to the economic dispatch principle, the lowest total cost will happen when Generator 1 and Generator 2 have the same incremental cost λ. P_1 and P_2 are the respective outputs of Generator 1 and Generator 2 corresponding to system incremental cost λ respectively, and $P_1 + P_2 = P_R$. If the energy market settles the electricity trade at the market-clearing price, obviously, the market-clearing price of the above example will be λ. Therefore, the users should pay Generator 1 and Generator 2 a total of:

$$\lambda * P_1 + \lambda * P_2 = \lambda * (P_1 + P_2) = \lambda * P_R$$

If the outputs of Generator 1 and Generator 2 deviate from P_1 and P_2 respectively, for example, Generator 1 backs its output from P_1 to P_1', Generator 2 raises its output from P_2 to P_2', and $P_1 + P_2 = P_1' + P_2' = P_R$, the incremental cost of Generator 1 will drop down to λ_1, and at the same time the incremental cost of Generator 2 will increase up to λ_2. In terms of the principle of non-discriminate auction executed in the energy market, the market-clearing price determined by the ISO will be λ_2. All accepted generators (here is Generator 1 and 2) will be paid at the price of λ_2. Therefore, the users should pay Generator 1 and Generator 2 a total of:

$$\lambda_2 * P_1' + \lambda_2 * P_2' = \lambda_2 * (P_1' + P_2') = \lambda_2 * P_R$$

Because $\lambda_2 > \lambda$, it indicates that users will pay more to generators if the generation schedule is not determined by traditional economic dispatch method. It means that traditional economic dispatch principle still applies to deregulated environment.

If users are not required to submit bidding prices to the market, the traditional economic dispatch method is enough to determine an optimal generation schedule. But many energy markets also require the users to bid for power demand. It increases the complexity of determining a generation schedule. A method that can be used to determine the optimal generation schedule if the customers are required to submit bidding prices is explained in the following paragraphs.

Suppose that a system has n generators and a group of customers (loads) who compete in the energy market. And, assume that each generator is required to submit a linear bid function to the ISO. Also assume that the *i*th generator's bid function is, $H(P_i) = \alpha_i P_i + \beta_i$, where P_i is the active output of Generator i and α_i and β_i are the the bid coefficients. A set of generation output limits, P_i^{max} and P_i^{min} (i=1, 2, …, n) is also specified by the generator. And, assume that the load function for a specified time period is:

$$P_R = Q_0 + Q(t) + Q(r); \quad t = 1, 2, 3, \ldots\ldots, 24$$

where, r is the market-clearing price.

Based on the bids and load function, the following equations can be written:

$$\alpha_i P_i + \beta_i = r \qquad (4.1)$$

$$\sum_{i=1}^{n} P_i = Q_0 + Q(t) + Q(r); \quad t = 1, 2, 3, \ldots\ldots, 24 \qquad (4.2)$$

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad i=1, 2, 3, \ldots, n$$

Equation (4.1) can be written as:

$$P_i = (r - \beta_i) / \alpha_i \qquad (4.3)$$

Substitute Equation (4.3) into Equation (4.2),

$$\sum_{i=1}^{n} (r - \beta_i)/\alpha_i = Q_0 + Q(t) + Q(r); \quad t = 1, 2, 3, \ldots\ldots, 24 \qquad (4.4)$$

Market clearing price, r can be solved from Equation (4.4). As soon as r is determined, Equation (4.3) can be used to determine the output of each generator. If the calculated output violates a generation output limit, it must be modified to accommodate the inequality constraint. When P_i is smaller than its lower limit $P_i^{\min}$, Generator i will face a net loss if price r is maintained. At the same time, if the price corresponding to $P_i^{\min}$ is paid to Generator i, the other generators will also be paid at the same price. Because the price corresponding to $P_i^{\min}$ is higher than r, the users will have to pay more money to generators. In this case, in order to set the output of Generator i, first set P_i to be $P_i^{\min}$. Evaluate the price $r1$ corresponding to $P_i^{\min}$. r1 is the assumed market-clearing price at which all generators should be paid. Next, set P_i to zero. It means that Generator i will be removed from the problem, and as a result, Generator i ceases to compete in the energy market. In this case, solving Equations (4.3) & (4.4), a new generation schedule and corresponding market clearing price $r2$ can be obtained. If $r2>r1$, P_i should be set to $P_i^{\min}$. Because, if P_i is set to zero, the market-clearing price is higher than when P_i set to $P_i^{\min}$. And if $r2<r1$, P_i should be set to zero rather than $P_i^{\min}$. If $r1=r2$, P_i could be chosen to set to zero or $P_i^{\min}$. However, it is better to set P_i to $P_i^{\min}$ unless this arrangement will cause congestion problem. It is because more generators put into operation means higher system reliability from the viewpoint of the ISO. When P_i

becomes larger than the upper limit, P_i should be set to P_i^{max}, and Equations (4.3) & (4.4) can be solved to determine a new market-clearing price and a generation schedule.

A simple system as shown in Fig. 4.8 is utilised to provide numerical example to explain the above-mentioned method. Bidding price function and output limit of each generator are listed in Table 4.1. Assume that there is a residential user at Node 6 and its load demand is 150 MW at 11:00 A.M. There is an industrial customer at Node 4 and its load demand follows a linear function: $Q(r) = 150 - 5*r$, where r is the market-clearing price.

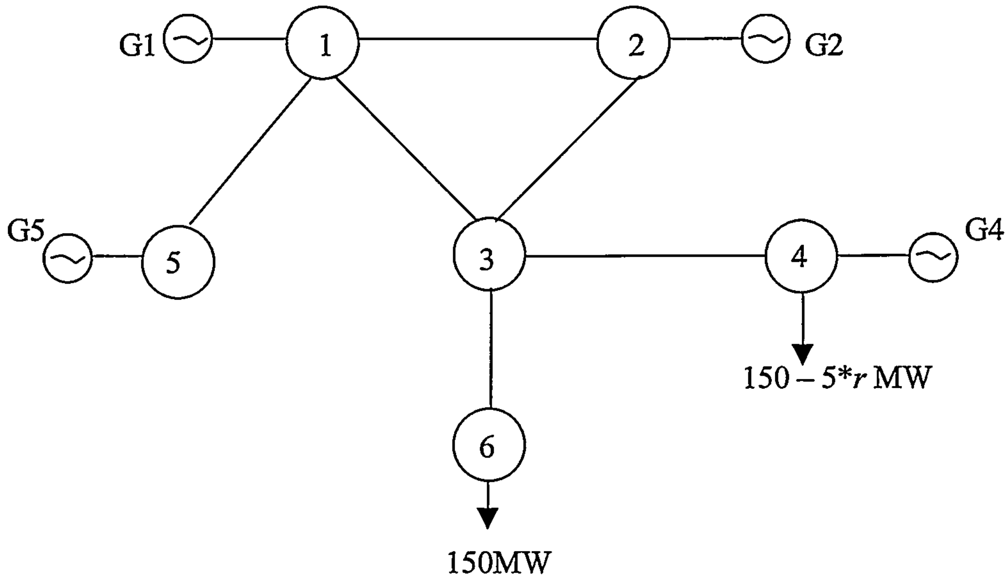


Fig 4.8 A System Used to Explain the Determination of the Optimal Generation Schedule in Deregulated Environment

Table 4.1: Generator Bidding Price Function

Bus	Unit	Bidding Price Function ($/MWh)	Max. Output (MW)	Min. Output (MW)
1	1#	$H = 0.048P_1 + 12.45$	100	40
2	2#	$H = 0.044P_2 + 14.36$	100	20
4	4#	$H = 0.058P_4 + 15.22$	120	20
5	5#	$H = 0.044P_5 + 10.65$	180	50

Let r be the market-clearing price. From the bidding price function, the active output of each generator can be expressed as:

$$\begin{aligned} &\text{G1} \quad P_1 = (r - 12.45)/0.048 = 20.833r - 259.375 \\ &\text{G2} \quad P_2 = (r - 14.36)/0.044 = 22.727r - 326.324 \\ &\text{G4} \quad P_4 = (r - 15.22)/0.058 = 17.241r - 262.414 \\ &\text{G5} \quad P_5 = (r - 10.65)/0.044 = 22.727r - 242.045 \end{aligned}$$

Because:

$$\sum_{i=1}^{n} P_i = P_R;$$

$$. \; P_R = 150 + 150 - 5r = 300 - 5r$$

and,

$$\begin{aligned} P_1 + P_2 + P_4 + P_5 &= (20.833r - 259.375) + (22.727r - 326.324) \\ &\quad + (17.241r - 262.414) + (22.727r - 242.045) \\ &= 83.528r - 1090.158 = P_R = 300 - 5r \\ r &= (1090.158 + 300)/(83.528 + 5) = \$15.703/\text{MWh} \end{aligned}$$

Therefore:

$$\begin{aligned} &\text{G1} \quad P_1 = 20.833r - 259.375 = 67.766 \text{ MW} \\ &\text{G2} \quad P_2 = 22.727r - 326.324 = 30.558 \text{ MW} \\ &\text{G4} \quad P_4 = 17.241r - 262.414 = 8.321 \text{ MW} \\ &\text{G5} \quad P_5 = 22.727r - 242.045 = 114.837 \text{ MW} \\ &P_R = 300 - 5r = 221.485 \text{ MW} \end{aligned}$$

From the above calculations, one can see that the active output of Generator 4 is only 8.321 MW that is lower than its limit of 20 MW. If the output of Generator 4 is set to 20 MW, the incremental cost of Generator 4 will be:

$$r = 0.058P_4 + 15.22 = 0.058*20 + 15.22 = \$16.38/\text{MWh}$$

Obviously, the market-clearing price will increase from \$15.703/MWh to \$16.38/MWh. Next, as a trial, remove Generator 4 from the competition and repeat the calculation process for active output and market clearing price:

$$P_1 + P_2 + P_5 = (20.833r - 259.375) + (22.727r - 326.324) + (22.727r - 242.045)$$
$$= 66.287r - 827.744 = P_R = 300 - 5r$$
$$r = (827.744 + 300)/(66.287 + 5) = \$15.820/\text{MWh}$$

Because \$15.820/MWh < \$16.38/MWh, the output of Generator 4 should be set to zero.

Finally,

$$\text{G1} \quad P_1 = 20.833r - 259.375 = 70.203 \text{ MW}$$
$$\text{G2} \quad P_2 = 22.727r - 326.324 = 33.217 \text{ MW}$$
$$\text{G5} \quad P_5 = 22.727r - 242.045 = 117.496 \text{ MW}$$
$$P_R = 300 - 5r = 220.9 \text{ MW}$$

In practice, no generator or customer will submit a linear bidding price function to the short-term energy market. They will bid for selling or buying electricity with different prices for different ranges of output or load demand. Fig. 4.8 shows a typical bidding price submitted by a generator.

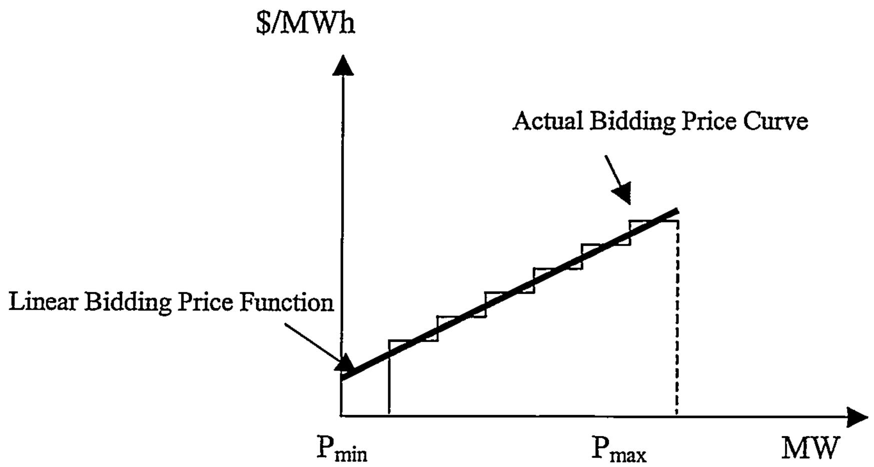


Fig. 4.9 Linear Bidding Price Function & Actual Bidding Price Curve

A typical bidding price curve is like a staircase. From it, one can obtain the underlying linear bidding price function as shown in Fig. 4.8. In a perfect competitive short-term market, actual bidding price curve of a generator will be very close to the linear bidding price function that perfectly reflects the incremental cost plus the profit rate. No generator is willing to play a risky game in a perfect energy market. Because they know that the market-clearing price is not determined by their own bids. In other words, an individual generator has no possibility to solely control the market-clearing price. If a generator bids too high, it is possible to lose the opportunity to sell its power. If it bids too low, it will face financial loss. Neither case is acceptable for the generator. In this case, a bidding price truly reflecting the generation cost plus reasonable profit is a preferable choice for a generator.

Therefore, as a system planner, when he or she considers the influence of short-term market to the transmission expansion planning, he or she should do the following:

1) Find out the load demand function for each customer under each typical condition, for example, the residential load demand function on a cloudy day in summertime, etc. This is a function of time and market-clearing price.
2) Find out the average number of days of each typical condition and the annual load increase rate for each customer.
3) Find out the price bidding function of each generator competing in the short-term energy market.
4) Using above functions, predict the possible generation schedule under each typical condition for short-term energy market.

5) Combining the predicted possible short-term generation schedule with the long-term generation schedule to calculate the system power flow distribution and possible congestion fee. And then, multiply the congestion fee with the average number of days of each typical condition
6) Add all the congestion fees under different typical conditions together to get the total congestion fee of a whole year.
7) Repeat the calculation process up to the usage life of a new transmission line to get congestion fees of different years, and then, sum them together to obtain the total congestion fee.
8) Finally, look for the optimal expansion planning. The investment cost of the new transmission line must be less than the total congestion fee.

A detailed numerical example to explain the calculation process mentioned above is presented in Chapter 5.

CHAPTER 5: A NUMERICAL EXAMPLE FOR ELECTRIC TRANSMISSION SYSTEM EXPANSION

A hypothetical system as shown in Fig. 5.1 is utilized in this chapter to provide a numerical example of including short-term power market factors into the transmission expansion planning.

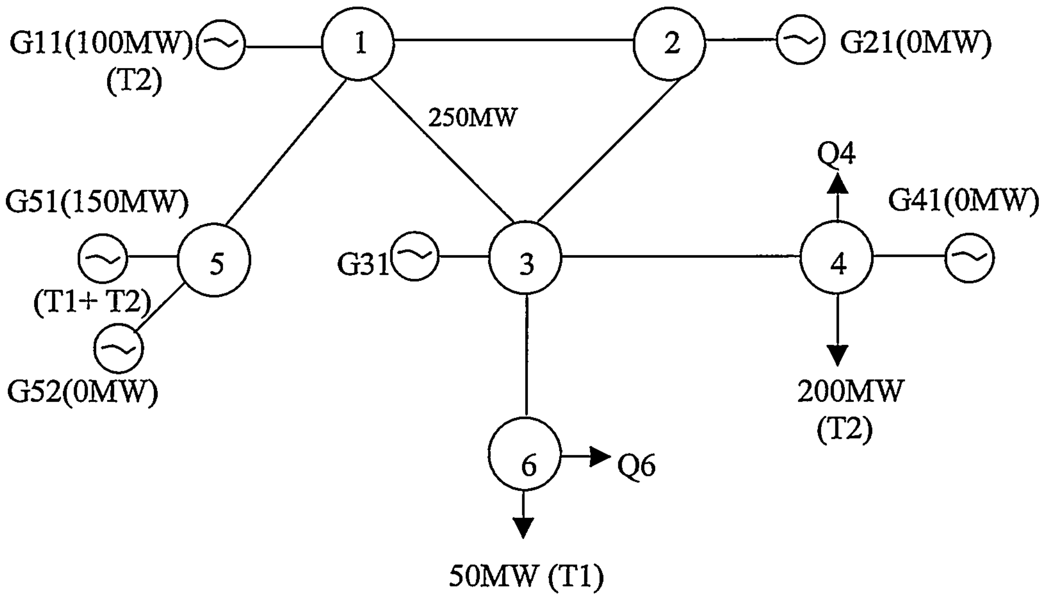


Fig. 5.1 A System Used to Illustrate the Calculation for Transmission Expansion

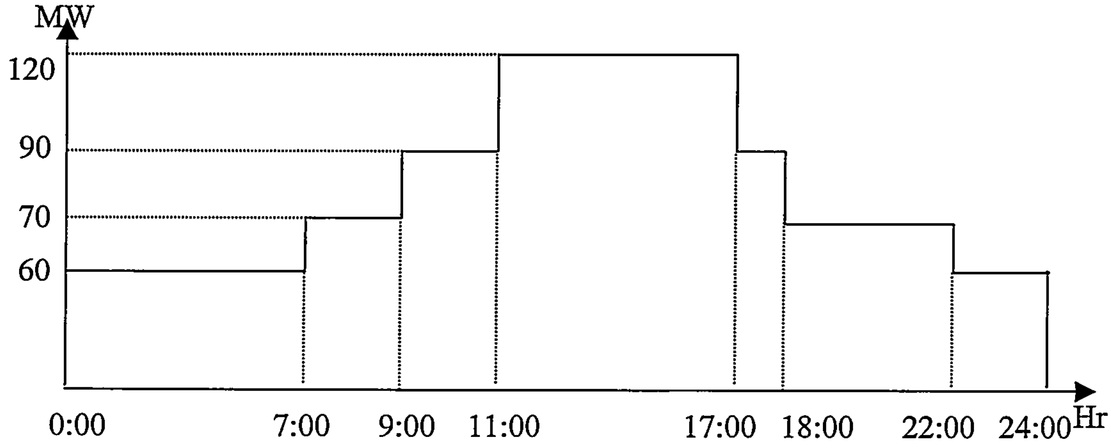
In this system, there are two long-term bilateral transactions: T1 and T2. The magnitudes of the transactions and the line flow limits are indicated in Fig. 5.1. As shown in the figure, Transaction T1 is scheduled to supply the load at Node 6 by Generator G51, and Transaction T2 is scheduled to supply the load at Node 4 by Generators G11 and G51 together. The remaining Generators G21, G41 and G52 (except G31) compete with each other in the short-term power market to supply the variable parts of the load: $Q4$ and $Q6$. It is assumed that all lines have the same resistance of 0.02 p.u and reactance of 0.1 p.u. The voltages at Nodes 1, 2, 3 and 5 are all 1.05 p.u. Loads 4 and 6 have the same power factor of 0.9. The base voltage of the system is 230 KV. Base MVA is 100MVA. Also assume that Generator G31 is used to compensate for the transmission loss and the price for loss compensation is $10/MWh. The generators' redispatch bid prices are listed in Table 5.1:

Table 5.1: Generator Redispatch Bid Price ($/MW)

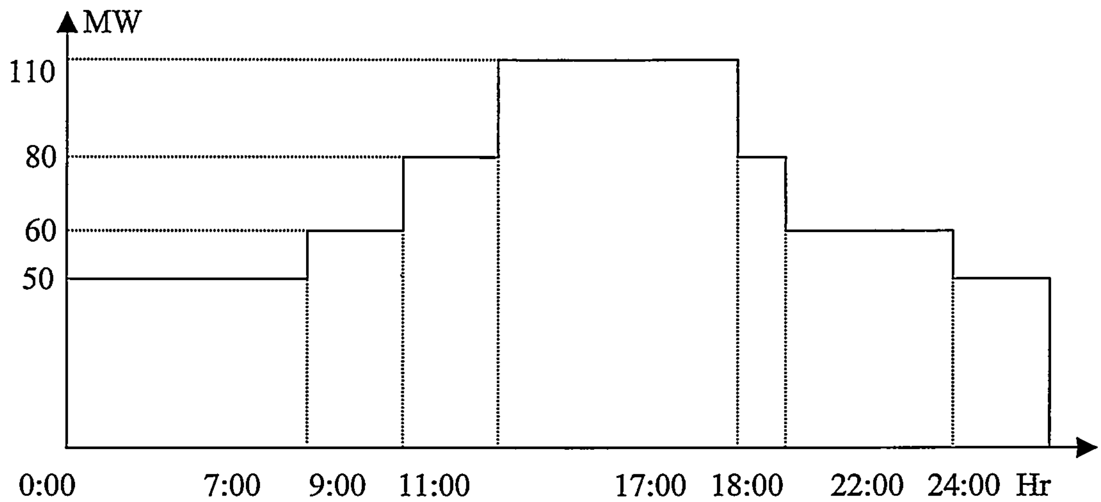
Bus	Generator	Output Up	Output Down
1	G11	+25	-15
2	G21	+30	-10
4	G41	+50	-10
5	G51	+40	-18
	G52	+40	-20

The calculation process has several steps which are illustrated in the following:

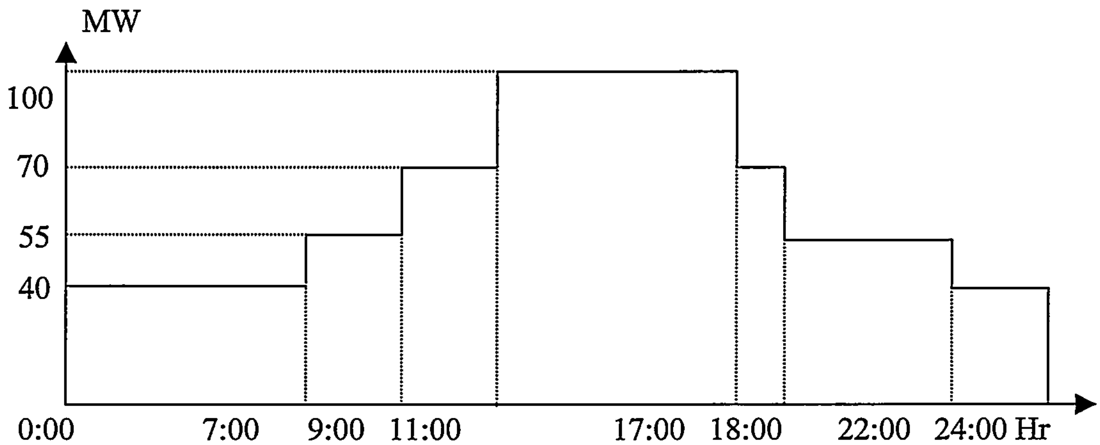
1) Find out the load demand function for each customer. In this example, it is assumed that there are two types of loads in this system. One is a combination of residential and commercial load, located at Node 6, and the other is an industrial load connected at Node 4. Assume that according to the analysis on the basis of historical load data submitted by the customers, the system planners have obtained the following typical load curves (as shown in Fig. 5.2), *Q6(t),* for the customer at Node 6:



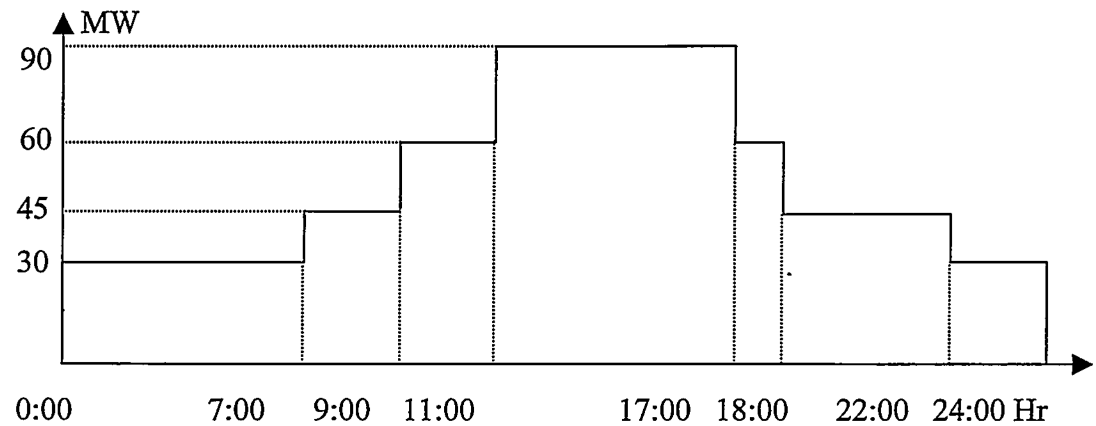
a) Typical Load Curve of Customer at Node 6 on a Cloudy Day in Winter



b) Typical Load Curve of Customer at Node 6 on a Sunny Day in Winter



c) Typical Load Curve of Customer at Node 6 on a Cloudy Day in Summer



d) Typical Load Curve of Customer at Node 6 on a Sunny Day in Summer

Fig. 5.2 Typical Load Curve

A typical industrial load consists of two parts: a non-price responsive part and a price responsive part. Assume that the non-price responsive part of the load at Node 4, *Q*4*(t),* has been found on the basis of the historical load data. *Q*4*(t)* is shown in Fig. 5.3.

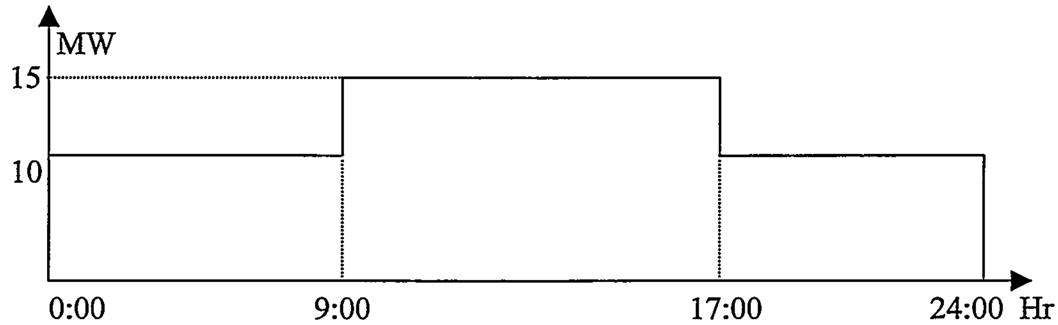


Fig. 5.3 Non-price Responsive Part of the Load at Node 4

Also assume that the price responsive part of the load at Node 4, *Q*4*(r),* is a straight line as shown in Fig. 5.4.

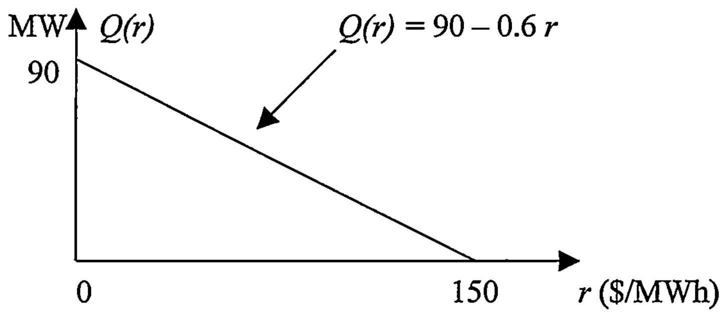


Fig. 5.4 Price Responsive Part of the Load at Node 4

The load at Node 4 can be expressed as:

$$Q4 = Q4(t) + Q4(r)$$

where, r is the market-clearing price ($/MWh).

2) Find out the average number of days for each typical weather condition and the annual load increase rate of each customer. Assume that the average number of days of each weather condition has been obtained from the historical weather statistics and is listed in Table 5.2.

Table 5.2: Average Days for Each Weather Condition (Days)

Winter		Summer	
Sunny	**Cloudy**	**Sunny**	**Cloudy**
100	80	140	45

It is assumed that the annual load increase rate of each customer is 3% for the customer at Node 6 and 2% for the customer at node 4. They are average load increase rates derived from the historical load growth of each customer.

3) Find out the bidding price function of each generator competing in the short-term power market. Although the bidding price function of each generator is private information owned by each power producer and the ISO cannot obtain it directly from the generation company, it can be acquired indirectly by the ISO through analysing historical bidding prices submitted by each generator. For this system, assume that the bidding price function of each generator competing in the short-term power market has been acquired through analysis and is listed in Table 5.3.

Table 5.3: Generator Bidding Price Function

Bus	Unit	Bidding Price Function ($/MWh)	Max. Output (MW)	Min. Output (MW)
2	G21	$H = 0.044P_2 + 24.36$	100	20
4	G41	$H = 0.058P_4 + 25.22$	120	20
5	G52	$H = 0.044P_5 + 20.65$	180	50

4) Obtain the generation schedule under each typical weather condition for the short-term power market. Using the calculation method introduced in Chapter 4, one can determine the generation schedule under each typical condition. The generation schedules for four typical weather conditions are shown in Tables 5.4 to 5.7 respectively. Market-clearing price for each hour is shown in the tables as well.

Table 5.4: Short-term Generation Schedule on a Cloudy Day in Winter (MW)

Hour	G21	G41	G52	MCP($/MW)	Variable Load at Bus 4	Variable Load at Bus 6
0:00	30.1351	0	114.4531	25.6862	60	84.5883
1:00	30.1351	0	114.4531	25.6862	60	84.5883
2:00	30.1351	0	114.4531	25.6862	60	84.5883
3:00	30.1351	0	114.4531	25.6862	60	84.5883
4:00	30.1351	0	114.4531	25.6862	60	84.5883
5:00	30.1351	0	114.4531	25.6862	60	84.5883
6:00	30.1351	0	114.4531	25.6862	60	84.5883
7:00	35.0700	0	119.3880	25.9034	70	84.4580
8:00	35.0700	0	119.3880	25.9034	70	84.4580
9:00	37.4268	20.0000	121.7448	26.3806	90	89.1716
10:00	37.4268	20.0000	121.7448	26.3806	90	89.1716
11:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
12:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
13:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
14:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
15:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
16:00	50.5916	23.5470	134.9096	26.5863	120	89.0482
17:00	44.9397	0	129.2577	26.3376	90	84.1974
18:00	35.0700	0	119.3880	25.9034	70	84.4580
19:00	35.0700	0	119.3880	25.9034	70	84.4580
20:00	35.0700	0	119.3880	25.9034	70	84.4580
21:00	35.0700	0	119.3880	25.9034	70	84.4580
22:00	30.1351	0	114.4531	25.6862	60	84.5883
23:00	30.1351	0	114.4531	25.6862	60	84.5883

Table 5.5: Short-term Generation Schedule on a Sunny Day in Winter (MW)

Hour	G21	G41	G52	MCP($/MW)	Variable Load at Bus 4	Variable Load at Bus 6
0:00	25.2003	0	109.5183	25.4691	50	84.7185
1:00	25.2003	0	109.5183	25.4691	50	84.7185
2:00	25.2003	0	109.5183	25.4691	50	84.7185
3:00	25.2003	0	109.5183	25.4691	50	84.7185
4:00	25.2003	0	109.5183	25.4691	50	84.7185
5:00	25.2003	0	109.5183	25.4691	50	84.7185
6:00	25.2003	0	109.5183	25.4691	50	84.7185
7:00	30.1351	0	114.4531	25.6862	60	84.5883
8:00	30.1351	0	114.4531	25.6862	60	84.5883
9:00	42.4723	0	126.7903	26.2291	80	89.2626
10:00	42.4723	0	126.7903	26.2291	80	89.2626
11:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
12:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
13:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
14:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
15:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
16:00	47.0010	20.8231	131.3190	26.4283	110	89.1430
17:00	40.0048	0	124.3228	26.1205	80	84.3277
18:00	30.1351	0	114.4531	25.6862	60	84.5883
19:00	30.1351	0	114.4531	25.6862	60	84.5883
20:00	30.1351	0	114.4531	25.6862	60	84.5883
21:00	30.1351	0	114.4531	25.6862	60	84.5883
22:00	25.2003	0	109.5183	25.4691	50	84.7185
23:00	25.2003	0	109.5183	25.4691	50	84.7185