RELIABILITY MODEL CONSTRUCTION AND EVALUATION IN REPAIRABLE SYSTEMS

A Thesis Submitted to
the Faculty of Graduate Studies and Research
in Partial Fulfilment of the Requirement for
the Degree of
Master of Science
in the
Department of Electrical Engineering
University of Saskatchewan

by

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June 1974

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ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. R. Billinton, who supervised this thesis, for his patient guidance and encouragement during the course of this work. Helpful discussions and suggestions by Dr. Channan Singh are also gratefully acknowledged. The appreciation is also extended to Mr. R. Popple and Mr. J.R. Irvings in Ontario Hydro for their comments and suggestions regarding the heat transport pump configuration.

This work was supported by a National Research Council Operation Grant.

UNIVERSITY OF SASKATCHEWAN

Electrical Engineering Abstract 74A161

"Reliability Model Construction and Evaluation

in Repairable Systems"

Student: S.Y. Lee Supervisor: Dr. R. Billinton

M.Sc. Thesis Presented to the College of Graduate Studies

June 1974

ABSTRACT

This thesis presents some basic techniques of reliability modelling and quantitative evaluation in repairable systems. The Markov approach is utilized to analyze the heat transport pump subsystem in the Bruce generating station of Ontario Hydro and the required state space models under the various assumptions are constructed by a computer program. The system availabilities and the value of spare pumps are evaluated from these models. The effects of permanent and temporary failures on the reliability indices are also examined. The Markov approach cannot be applied directly if the distributions associated with all the system contingencies are not exponentially distributed. Reliability evaluation of a non-Markovian system is usually much more complicated than that of a Markovian system. A practical non-Markovian technique, the method of stages, is discussed in this thesis and characteristics of several basic stage combinations are presented. The method is applied to a practical system and the results are compared with those of other techniques. The simulation method, which is also popular in reliability evaluation, is illustrated by application to various practical problems. The efficiency of simulation can be improved by developing an accelerated simulation method and an example of this method is presented in this thesis.

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1. INTRODUCTION

A basic requirement of a modern electric power system is the ability to constantly supply power to the customer. This ability can be quantitatively measured and predicted.[1] Reliability evaluation is becoming an important aspect in economic analysis of power systems. Qualitative assessment of system performance is no longer adequate because of the rapidly increasing demand for electricity and the growing concern over the environment. Reliability evaluation is an essential part of overall power system analysis in the areas of planning, design and operation.[2,3] Reliability is an inherent property of a system. A desired level of reliability must be met at the design stage because it cannot be easily altered at some later time. A high level of system reliability is always desirable, but it may not be economically feasible. The economic implication of reliability must be fully appreciated in the design stage. A suitable criterion of adequate performance is a matter of engineering appraisal and system appreciation. This level of adequacy must be decided by management in terms of continuity or quality of service. The average system performance over a relatively long time can be predicted using numerical reliability techniques. These predictions must be incorporated in the design process, if the desired target is to be achieved. Reliability evaluation also serves as a means of obtaining a consistent quantitative assessment of several proposals and a means of selecting the best design for the selected proposal.

Consistent techniques are required for evaluating numerical reliability indices in the various areas of application. There has been a large number of publications concerning the reliability concepts and evaluation techniques (reference 1). Probability concepts are the basic tools used for quantitative reliability evaluation. There still exists some reluctance to accept this approach because the indices are not of deterministic nature and because of the lack of confidence in the required data. The probability approach is, however, the only consistent method for quantitative reliability evaluation and one which can successfully replace the old rule of thumb approaches. The calculated indices are the best values which can be obtained with the available data. It is only a matter of improving the data collection method to attain a higher confidence level in the basic component data.

Probability techniques have been applied to many areas of application within existing and proposed power systems.[2] A large quantity of work has been performed on the evaluation of the quantitative capacity requirements of a generation system. There are two basic areas of generation system reliability evaluation. The adequacy of the total planned or installed capacity of the system in regard to satisfying the load is the primary concern in static generating capacity reliability evaluation. The risk level at which the load will exceed the probably available capacity is predicted. The second basic area is one in which the main concern is the determination of the ability of the system to meet the contingent unforeseen load conditions in actual operation. The techniques used in transmission system reliability evaluation are basically similar to those used in the generation system field except that more reliability parameters are required to sufficiently assess the system adequacy. Composite system reliability indices can be obtained from the generation and transmission system reliability components. These techniques are also applied in other system or subsystem areas. The assessment of component redundancy is a particularly important consideration in reliability evaluation of subsystems.(6)

There have been many analytic techniques proposed to evaluate reliability indices for various power systems or subsystems. These techniques utilize probability theory as a basic mathematical tool. The techniques are usually well-suited for small and uncomplicated systems, but cannot be applied in large and complicated practical systems. A widely used technique, the Markov approach, can be applied to a relatively large system, but is also limited by the system size. Some attempts have been made recently to overcome this difficulty, either in the form of approximate methods or by developing methods of utilizing electronic digital computers to take over laborious computing procedures.(4) Some interest has also been shown in non-Markovian processes. These methods are, in general, improvements of the existing techniques.

The work which has culminated in this thesis was primarily directed towards developing improved techniques for reliability evaluation. Determination of reliability indices from a Markovian model is relatively straightforward by using existing techniques. On the other hand, the construction of a large model is usually time-consuming and impractical. This is the main disadvantage of using the Markov approach in practical applications. A general method of constructing Markov models for large systems is proposed in this thesis and applied systematically to the heat transport pump system in the Bruce generating station. This approach is general and can be applied to other similar problems. Mathematical analysis of a non-Markovian model is very difficult, if at all possible. Some techniques have been proposed in the past, but are very limited in their application. An approximate method of transforming a non-Markovian process into an equivalent Markov model is presented in this thesis (reference 17). This approach has been applied to a generation system model with non-exponential down times. If all analytical techniques fail in the case of a particular problem, a simulation method can be tried. The simulation method has been discussed in Chapter 4 of this thesis, applied to some example systems and compared with the analytical methods.

2. MARKOVIAN MODEL OF HEAT TRANSPORT PUMPS

2.1 Introduction

As indicated in the introduction to this thesis, the theoretical concepts of the techniques for model building and solution were developed by application to a practical system problem. Information was provided by Ontario Hydro on the configuration of the heat transport circulating pump system at their Bruce generating station. This system is, therefore, used as a practical framework for the development and investigation of alternate model forms.

The Bruce generating station contains four nulcear power units each of which is powered by a nuclear reactor. The net electrical power output for each unit is approximately 750MW. The reactor is moderated by heavy water and cooled by pressurized heavy water. The reactor coolant is circulated by four heat transport pumps. The heat is then released in the boiler. Failure of any one of the four pumps in a unit reduces the power output by 25 percent. The failures of two heat transport pumps results in the complete shut-down of the unit. Failures of heat transport pumps in operation are assumed to be independent of operational modes of other components.

Failure data for large pumps were obtained from Ontario Hydro and are shown in Table 2.1. The failures are grouped into one or two failure modes in an actual reliability analysis and each group of failures is represented by a single failure rate. Three different

Table 2.1: Failure Data for Large Pumps

Service	Estimated Fraction of Failures and Average Downtime Caused by Various Failure Modes						Failure/unit Year and Average Downtime		
	Bearing	Motor Winding	Motor Cooling	Pump Seals	Erosion Cavities	Other	Ontario Hydro Conventional	Ontario Hydro Nuclear	Edison Electric Institute
Boiler	12%	12%	10%	6%	10%	50%	.9	1.1	0.8
Feed Pumps (1000-4500HP)	30h	30h	8h	8h	500h (est)	100h	110h	-	10h
Condensate Service Water	7%	15%	3%	5%	5%	65%	.7	1.0	.3
Air Pumps (300-800HP)	30h	40h	8h	8h	500h (est)	50h	70h	-	2h

classification methods of failure modes are used in this thesis: 1) single failure mode assumption where all failures are grouped into one permanent failure mode, 2) mixed failure modes assumption where all failures are grouped into a temporary and a permanent failure modes, 3) two failure modes assumption where all failures are grouped into two different permanent failure modes. A reliability study of the heat transport pump system is simplest under the single failure mode assumption. This assumption was chosen for the major part of the work in this chapter. Reliability models under the other two assumptions were also examined and compared to the model of a single failure mode.

The models are complicated because a heat transport pump operation depends on the other pump failures. The failure rate of pumps not in operation was assumed to be negligible. If two pumps in a unit failed, no further pump failures would occur because the unit would be shut down. Failures of the remaining two pumps, therefore, depends on the performance of other pumps. The pumps in a unit must be considered collectively in reliability studies due to this dependency. There exists no such dependency between two pumps in separate units. This independency disappears if spare pumps are shared between units.

Distributions associated with repair and installation times are assumed to be exponential. The time between failures is also assumed to be exponentially distributed. The random process associated with the reliability model for the pump system becomes Markovian under these assumptions. Analysis of this type of process is relatively easy and well systematized.[2,5] If all the distributions are not exponential, the process becomes non-Markovian. Analysis of a non-Markovian process is in general much more complicated than that of a Markov process. Non-exponential aspects of down time distributions are discussed in the next chapter and a method for modelling non-Markovian process is proposed.

A reliability model for the heat transport pump system is large and complicated. Construction of the entire state space model is, therefore, very difficult and time consuming. A computer technique was developed for state space model constructions and was applied to the heat transport pump system to obtain the materials presented in this chapter. A precise method of representing states was required to identify a large number of states.

Bruce generating station is designed to operate as a base load station and therefore only the long term reliability of the station is considered in this thesis. Only the impact of heat transport pump outages on the power output of the station has been considered and all other components are assumed to operate perfectly without failures.

2.2 The Heat Transport Pump Models Under A Single Failure Mode Assumption

All possible pump failures are grouped into a single permanent failure mode. Every permanent failure results in the pump removal from the operating site to the repair shop. It is assumed that every permanently failed pump can be repaired and put back in operation. Two types of action are required for the pump restoration: repair and installation. The following symbols are utilized throughout the section:

λ = permanent failure rate of an operating pump

μ = repair rate of a permanently failed pump

γ = installation rate of a spare pump.

The three rates are constant with respect to time under the assumptions of the Markovian process discussed in the introduction to this chapter.

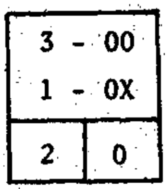
2.2.1 Method of State Description

There are many possible methods for representing a state of the heat transport pump system. The state description method used in this chapter divides the plant into three sections: the operating site, the repair shop and the warehouse. The states of the plant are described by state representations of its three different sections. The operating site state provides the information in connection with power unit operations. The repair shop state provides information about the failed pumps undergoing repair. The warehouse state describes the situations of the stored pumps.

The operating site state is described by state representations of the power units within it. Each power unit has less than or equal to four pumps. State representation of the unit consists of the state representation of these pumps. State representation of two of those pumps can be left out because at least two pumps are available in a unit at any time due to the assumption that a pump is liable to failure only during its operation. There are two possible types of pumps on the operating site: operating pump and pump undergoing repair. These pumps are represented by the symbols 0 and I, respectively, in state space diagrams. A pump deficiency in a power unit is represented by the symbol X in state space diagrams. A unit has six possible states:

state 1, where all pumps are operating (represented by 00 in state space diagrams); state 2, where one pump has been removed (represented by 0X); state 3, where an installation is in progress (represented by 0I); state 4, where two pumps have been removed (represented by XX); state 5, where one pump has been removed and an installation is in progress (represented by IX); state 6, where two installations are in progress (represented by II).

The repair shop state can be described sufficiently by the number of pumps undergoing repair, and the warehouse state by the number of stored spares. For example, the state of the plant, where three units are in full operation and a pump has been removed from the other unit and two pumps are undergoing repair and no spare is in the warehouse, is shown in the state space diagram as follows:



The upper part of the block contains the information for the operating site state; the lower left part for the repair shop state; and the lower right part for the warehouse state.

2.2.2 Single Unit Model

The reliability modelling technique is applied to a single unit case. The state space diagram of this model is shown in Figure 2.1. There is no spare provided in the system. The maximum number of failed pumps in the repair shop is two because no more than two failures are possible in a unit. Since the number of pumps required in the operating

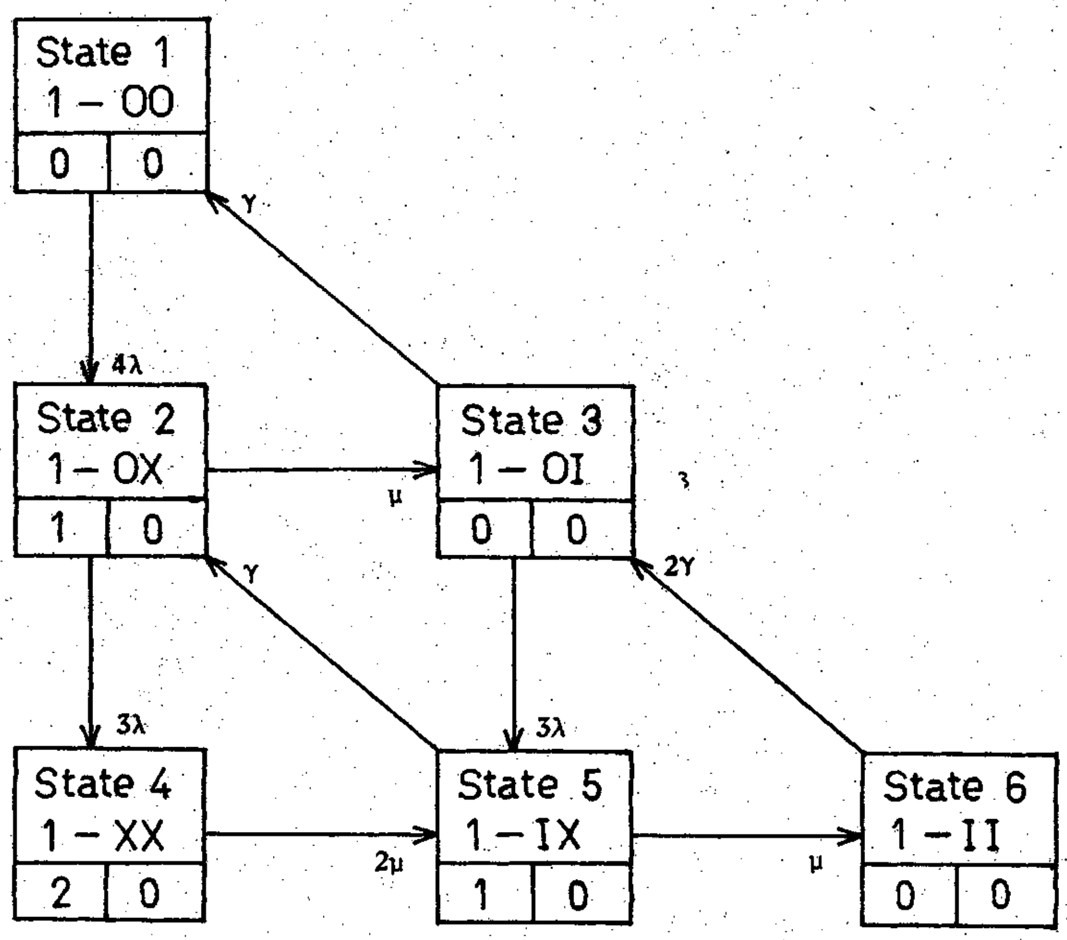


Figure 2.1: State Space Diagram of a Single Unit with no Spare Pump.

site is equal to the total number of pumps in the plant, no idle spare is stored in the warehouse at any time. The warehouse is not needed in this case for the purpose of storing excess spare pumps.

A listing of all possible states and transitions between them can be used to describe the single unit model as shown in Table 2.2 instead of the state space diagram. No information is omitted in the table. This form of expression is sometimes more convenient than the state space diagram. Advantage of the diagram disappears when a simple geometric pattern cannot be found within it. The table type is more convenient for large and complex models.

Steady state probabilities and frequencies for the states can be calculated by solving the linear simultaneous equations associated with the system model. The equations can be set up by using the fact that the frequency of entering a set of states is equal to the frequency of leaving it in the steady state condition. Choosing the five sets of states, {state 1}, {state 4}, {state 6}, {states 1,2,4}, {states 4,5,6}, and equating the frequency of entering each set to the frequency of leaving it, the following equations can be obtained:

$$4\lambda P_1 = \gamma P_3 \qquad \text{............... (2.1)}$$

$$3\lambda P_2 = 2\mu P_4 \qquad \text{............... (2.2)}$$

$$\mu P_5 = 2\gamma P_6 \qquad \text{............... (2.3)}$$

$$\mu P_2 + 2\mu P_4 = \gamma P_3 + \gamma P_5 \qquad \text{............... (2.4)}$$

$$3\lambda P_2 + 3\lambda P_3 = \gamma P_5 + 2\gamma P_6 \qquad \text{............... (2.5)}$$

There are $2^5 - 1$ different ways of selecting a set in this case, but only

Table 2.2: Table of the Single Unit Model Without a Spare

State Identification

State Number	00	0X	0I	XX	XI	II	Repair Shop	Warehouse
1	1	0	0	0	0	0	0	0
2	0	1	0	0	0	0	1	0
3	0	0	1	0	0	0	0	0
4	0	0	0	1	0	0	2	0
5	0	0	0	0	1	0	1	0
6	0	0	0	0	0	1	0	0

Transition Rates

States		
From	To	Rate
1	2	4λ
2	3	μ
3	1	Υ
2	4	3λ
3	5	3λ
4	5	2μ
5	2	Υ
5	6	μ
6	3	2Υ

five linearly independent equations can be obtained. The following equation must be included:

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1.0 \quad \ldots\ldots\ldots\ldots\ldots (2.6)$$

The state probabilities were obtained from the above equations:

$$P_1 = \mu^2\gamma^2/A, \quad P_2 = 4\lambda\mu\gamma^2/A, \quad P_3 = 4\lambda\mu^2\gamma/A,$$

$$P_4 = 6\lambda^2\gamma^2/A, \quad P_5 = 12\lambda^2\mu\gamma/A, \quad P_6 = 6\lambda^2\mu^2/A,$$

$$\text{where } A = 6\lambda^2(\mu + \gamma)^2 + 4\lambda\mu\gamma(\mu + \gamma) + \mu^2\gamma^2.$$

The frequency of encountering each state is calculated by multiplying the state probability and the rate of departure from the state:

$$f_1 = 4\lambda\mu^2\gamma^2/A, \quad f_2 = 4\lambda\mu\gamma^2(3\lambda + \mu)/A, \quad f_3 = 4\lambda\mu^2\gamma(3\lambda + \gamma)/A,$$

$$f_4 = 12\lambda^2\mu\gamma^2/A, \quad f_5 = 12\lambda^2\mu\gamma(\mu + \gamma)/A, \quad f_6 = 12\lambda^2\mu^2\gamma/A.$$

Steady state reliability indices for the no spare case were calculated substituting the following values and are listed in Table 2.3: $\lambda = 0.6$ failures/year, $\mu = 35.04$ repairs/year, $\gamma = 292$ inst./year. Reliability indices were also calculated for the different capacity states and shown in Table 2.3.

Availability of the unit can be improved by providing spare pumps. The number of states increase with the addition of spares as shown in Appendix 1. A state space diagram for the case of a single spare is shown in Figure 2.2. The warehouse is required for possible storage

Table 2.3: Steady-state Probabilities, Frequencies and Average Durations For the States of the Single Unit Model Without Spares

State Elements

State	Probability	Frequency (per year)	Average Duration (year)
1	0.9268535	2.2244485	0.4166667
2	0.0634831	2.3387181	0.0271444
3	0.0076180	2.2381608	0.0034037
4	0.0016306	0.1142696	0.0142694
5	0.0003913	0.1279820	0.0030577
6	0.0000235	0.0137124	0.0017123

Capacity States

Capacity	Probability	Frequency (per year)	Average Duration (year)
4	0.9268535	2.2244485	0.4166667
3	0.0711011	2.3524304	0.0302245
0	0.0020454	0.1279820	0.0159817

Incapability factor = 0.0198208

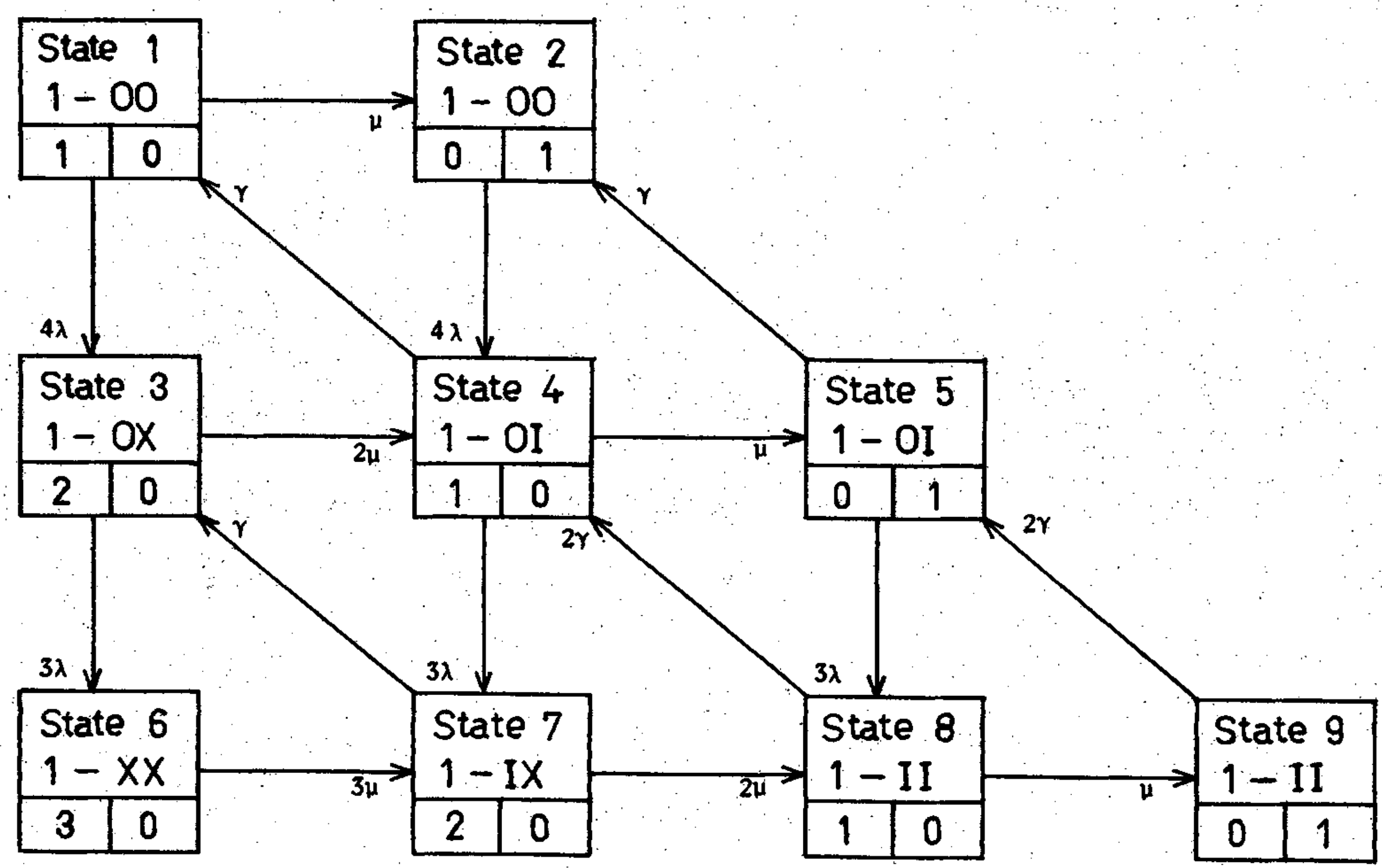


Figure 2.2: State Space Diagram of a Single Unit with one Spare Pump

of spare pumps. The maximum number of pumps in the warehouse is the same as the number of spares initially provided in the system. State probabilities were found by constructing the transition matrix of the model. Capacity state probabilities are also found from the state probability values and are listed in Table 2.4. Calculations of the capacity state probabilities are repeated for various numbers of spare pumps and the results are also listed in Table 2.4.

A single reliability index, designated as incapability factor (I.F.), has been used in this chapter. The incapability factor is defined as the ratio of the energy production lost due to pump outages of a unit or a station to the perfect net output for the period of time considered. The I.F. can be obtained from the capacity state probabilities as follows for the single unit case:

$$\text{I.F.} = 0.25 \times P\{75\% \text{ of full power output}\} + 1.0 \times P\{\text{unit down}\}.$$

The following equation can be used for more general cases:

$$\text{I.F.} = \sum_{i=1}^{n} \left(\frac{C-C_i}{C}\right) P_i ,$$

where C is the full power output of a system, n is the total number of different capacity states, C_i is the power output for ith capacity state, P_i is the probability of the ith capacity state. Incapability factors were calculated for the one unit case as the number of spares are varied and are listed in Table 2.4.

As the number of spares increases, the system becomes more available until it reaches a certain limit. If there are enough

Table 2.4: Capacity State Probabilities and Incapability Factors as the Number of Spare Pumps is Varied

	PROBABILITY					
Capacity	Spare 0	Spare 1	Spare 2	Spare 3	Spare 4	Spare Infinite
100%	0.9268535	0.9896941	0.9917778	0.9918222	0.9918229	0.9918230
75%	0.0711011	0.0102156	0.0081953	0.0081526	0.0081520	0.0081519
0	0.0020454	0.0000904	0.0000269	0.0000252	0.0000251	0.0000251
	1.0	1.0	1.0	1.0	1.0	1.0
Incapability Factor	0.0198208	0.0026444	0.0020756	0.0020632	0.0020632	0.0020632

spares in the system, there will always be some spares in the warehouse to replace a failed pump. Only installation is required for restoration. This is equivalent to having zero repair time. The system availability cannot be improved to 100 percent by addition of spares because the restoration time for a failed pump cannot be reduced less than the installation time. In the hypothetical infinite spare case, the repair shop and the warehouse have little meaning. A state space diagram for the infinite spare case is shown in Figure 2.3. Consideration of the infinite spare situation is very useful when evaluating the limit in the incremental benefits of additional spares.

The steady state probabilities can be calculated from the model shown in Figure 2.3. They can also be calculated by letting the repair rate approach infinity in the expression of the state probabilities for the no spare case. As μ approaches infinity, the state probabilities will have the following values:

$$P_1 = \gamma^2/B, \quad P_2 = 0, \quad P_3 = 4\lambda\gamma/B, \quad P_4 = 0,$$

$$P_5 = 0, \quad P_6 = 6\lambda^2/B, \quad \text{where} \quad B = 6\lambda^2 + 4\lambda\gamma + \gamma^2.$$

The probabilities of states requiring spares become zero. Capacity state probabilities and the incapability factor for the infinite spare case were obtained and listed in Table 2.4.

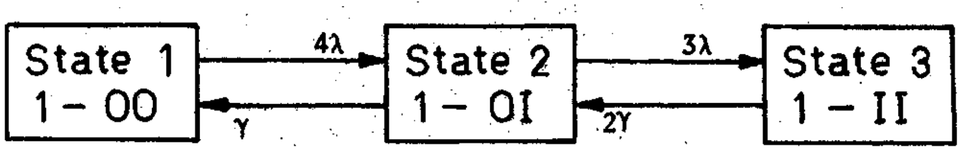


Figure 2.3: State Space Diagram of a Single Unit
Assuming an Infinite Number of Spares

2.2.3 Frequency of Event Occurrence

Frequencies of some events may be evaluated for the heat transport pump system besides the frequencies of state occurrences which have already been considered in the previous section. The frequency of occurrence of an event E is obtained by the following formula:

$$f_E = \sum_{i \in S} \lambda_i P_i$$

where f_E is the frequency of occurrence of the event E, S is the set of states in which the event E can possibly occur, and λ_i is the total rate of occurrence of the event E from the state i. The frequency of pump failure for the case of single unit without spare (Figure 2.1), therefore, is given by:

$$f_F = 4\lambda P_1 + 3\lambda P_2 + 3\lambda P_3$$

$$= (4\lambda\mu^2\gamma^2 + 12\lambda^2\mu\gamma^2 + 12\lambda^2\mu^2\lambda)/A$$

where $A = 6\lambda^2(\mu + \gamma)^2 + 4\lambda\mu\gamma(\mu + \gamma) + \mu^2\gamma^2$. The frequency of pump repair is given by:

$$f_R = \mu P_2 + 2\mu P_4 + \mu P_5 = f_F.$$

The frequency of pump installation is given by:

$$f_I = \gamma P_3 + \gamma P_5 + 2\gamma P_6 = f_F.$$

The three frequencies are identical. It is also true for any other heat transport pump configuration under the steady state condition.

If a pump fails on the operating site, it is not immediately accessible for the pump removal. The unit must be shut down and the coolant must be cooled, depressurized and drained before the pump can be isolated and removed. The start up of the unit from the cooled off state takes the reverse process of the shut down. Both processes are assumed to require eight hours. The shut down and start up times further reduce the system availability. This effect depends on the frequencies of the following events:

E1) A pump failure in a unit operating at full power.

E2) Completion of a pump installation in a unit operating at 75 percent of the full power.

E3) A pump failure in a unit operating at 75 percent of the full power.

E4) Completion of a pump installation in a unit shut down due to pump outages.

Both events, E_1 and E_2, result in shut down and restart of the unit. Sixteen hours are required for the process. The event E_3 results in the unit shut down which requires eight hours. The event E_4 results in the unit restart which requires eight hours. Total down time per year due to these time factors can be obtained by:

$$T = (16\text{ hours}) \times (\text{frequencies of occurrences of } E_1 \text{ and } E_2) + (8\text{ hours}) \times (\text{frequencies of occurrences of } E_3 \text{ and } E_4),$$

or more precisely by:

$$T' = 8760T / (T + 8760).$$

The increase in the incapability factor is:

$$T' (1.0 - \text{incapability factor}) / 8760.$$

The frequencies of events are calculated for the single unit system as the number of spares is varied, and are listed in Table 2.5. The frequency of failure increases as the number of spares increases. It will always be less than 4λ (= 2.5 failures per year). The increase in the incapability factor due to start up and shut down times is also listed in Table 2.5. The value is high and relatively constant.

2.2.4 Four Unit Model

If there is more than one unit in the plant, the installation of a spare can take place in several ways depending on the situation. A unit will require a spare for installation if it is in one of the states, 0X, IX, XX. Sometimes only one spare may be available when there is more than one unit in those states. Installation to a unit in a derated state results in a power output increase by ¼ x 750MW, while to a down unit by 750MW. A decision policy for spare assignment must be established for consistent reliability model construction. There exists six possible policies: a spare pump is delivered for installation to units in the order of being in the states; (1) XX, IX, 0X; (2) XX, 0X, IX; (3) IX, XX, 0X; (4) IX, 0X, XX; (5) 0X, XX, IX; (6) 0X, IX, XX. The unit in the state, which is earliest among the existing states of all units in the plant in the order of priority chosen, will first receive the spare pump.

In order to examine the consequence of a particular choice of policy, the incapability factor was calculated for all six cases and

Table 2.5: Frequencies of Event Occurrences and Increase in the Incapability Factor Due to Start Up and Shut Down Times

Number of Spare Pumps	Frequency (per year)							Increase in the Incapability Factor
	Failure	Repair	Installation	E1	E2	E3	E4	
0	2.35243	2.35243	2.35243	2.22444	2.22444	0.12798	0.12798	0.0081260
1	2.39365	2.39365	2.39365	2.37526	2.37526	0.01838	0.01838	0.0086123
2	2.39501	2.39501	2.39501	2.38026	2.38026	0.01475	0.01475	0.0086286
3	2.39504	2.39504	2.39504	2.38037	2.38037	0.01467	0.01467	0.0086290

listed in Table 2.6. No spares were assumed initially in the system. The priority sequence has no effect on the reliability for the single unit case. The incapability factors vary as the number of units is changed, because operation of more than one unit becomes dependent when the required components are apportioned using a given consistent decision policy. If independent unit operation is assumed, the incapability factor value must remain constant regardless of the number of units. The I.F. value shows a decrease for cases (1), (2) and (3) in Table 2.6, and an increase for cases (4), (5) and (6). The state space diagram of a two unit system operating without spares is shown in Figure 2.4 as an example of model construction adopting a constant policy.

A four unit system model without spares has 126 states as shown in Appendix 1. The number of states increases as more spares are added to the system. Availabilities of different capacity levels are calculated by truncating low probability states and are listed in Table 2.7. The incapability factors for different cases of spare provisions were also calculated and listed in Table 2.7. Addition of the first spare to the system shows the greatest decrease in the incapability factor. This benefit decreases as the number of spares increases. The state space diagram for four units with infinite spares is shown in Figure 2.5. The capacity state probabilities and the incapability factor were evaluated from the model and listed in Table 2.7. The incapability factor for four units is equal to that for a single unit in the case of infinite spares. The capacity state probabilities can also be evaluated from those for a single unit by a simple combination method.

Table 2.6: Incapability Factors as the Installation Priority Changes

		No. of Units			
No.	Priority	1	2	3	4
1	XX-IX-0X	0.0198208	0.0197956	0.0197723	0.0197500
2	XX-0X-IX	0.0198208	0.0197974	0.0197756	0.0197549
3	IX-XX-0X	0.0198208	0.0197960	0.0197729	0.0197511
4	IX-0X-XX	0.0198208	0.0198680	0.0199161	0.0199648
5	0X-XX-IX	0.0198208	0.0198680	0.0199159	0.0199642
6	0X-IX-XX	0.0198208	0.0198684	0.0199167	0.0199655

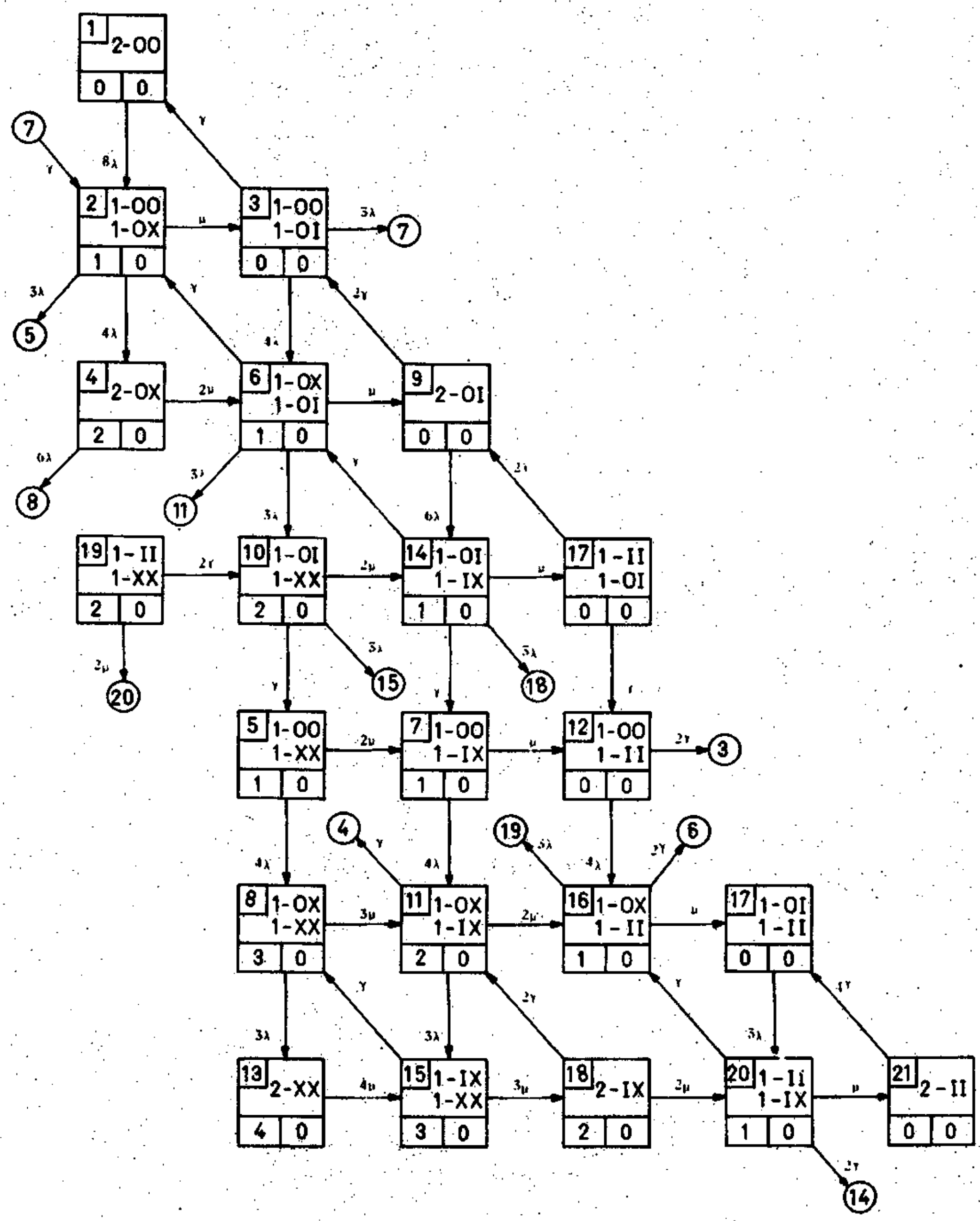


Figure 2.4: State Space Diagram of Two Units without Spares.

Table 2.7: Capacity State Probabilities and Incapability Factors For Four Units Assuming a Single Failure Mode

	Probability					
Number of Pumps in Operation	Spare 0	Spare 1	Spare 2	Spare 3	Spare 4	Infinite Spares
16	0.7379742	0.9380839	0.9651214	0.9675240	0.9676821	0.9676906
15	0.2264469	0.0577728	0.0340788	0.0319601	0.0318219	0.0318145
14	0.0265561	0.0031562	0.0006246	0.0004082	0.0003931	0.0003922
13	0.0014113	0.0001116	0.0000100	0.0000027	0.0000022	0.0000021
12	0.0060438	0.0007498	0.0001538	0.0001019	0.0000983	0.0000981
11	0.0014267	0.0001169	0.0000108	0.0000030	0.0000025	0.0000024
10	0.0001147	0.0000071	0.0000004	0.0	-	-
9	0.0000031	0.0000002	0.0	0.0	-	-
8	0.0000199	0.0000013	0.0000001	0.0	-	-
7	0.0000033	0.0000002	0.0	0.0	-	-
	1.0	1.0	1.0	1.0	1.0	1.0
I.F.	0.0197500	0.0042537	0.0022519	0.0020755	0.0020638	0.0020631

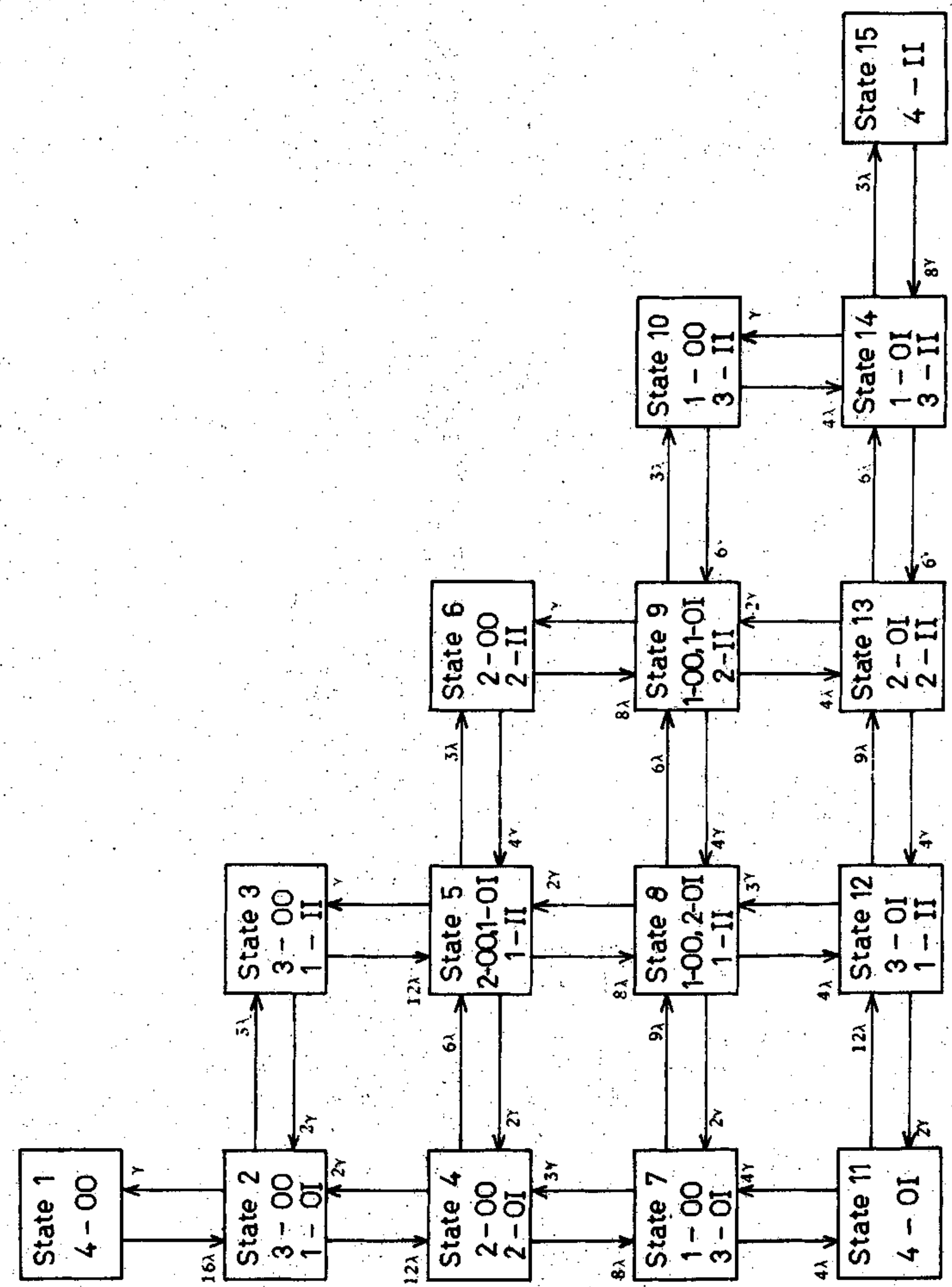


Figure 2.5: State Space Diagram of Four Units Assuming an Infinite Number of Spares.

In the previous studies, the values of failure, repair and installation rates were fixed. The incapability factors have been calculated for cases in which the three rates are varied. An increase in the failure rate produces an increase in the incapability factors as shown in Figure 2.6. The effect of the repair rate on the incapability factor is shown in Figure 2.7. In each case, only one rate is varied. Change in the installation rate has little effect on the incapability factor as shown in Figure 2.8.

The value attributed to a spare is defined in this thesis as the resulting decrease in the incapability factor due to the spare addition. The change produced by the addition of the first spare is plotted against the original incapability factor in Figure 2.9 as the transition rates are varied. Each curve shows a positive slope and indicates that the benefit from the first spare is greater in a system with a lower reliability. If reliability analysis of the system is done with optimistic parameter values, the worth of a spare is underestimated. If pessimistic parameter values are used, the worth of a spare tends to be overestimated.

The reliability evaluations so far discussed have been performed assuming infinite repair capability. If the number of pumps that can be repaired at a given time cannot exceed a certain value, the system availability will decrease. The incapability factors under repair restrictions are listed in Table 2.8.

2.2.5 Equivalent Model of a Single Pump in Four Unit Operation

An equivalent model of a pump operating as an element in the plant is considered. A pump operation is shown in Figure 2.10. There

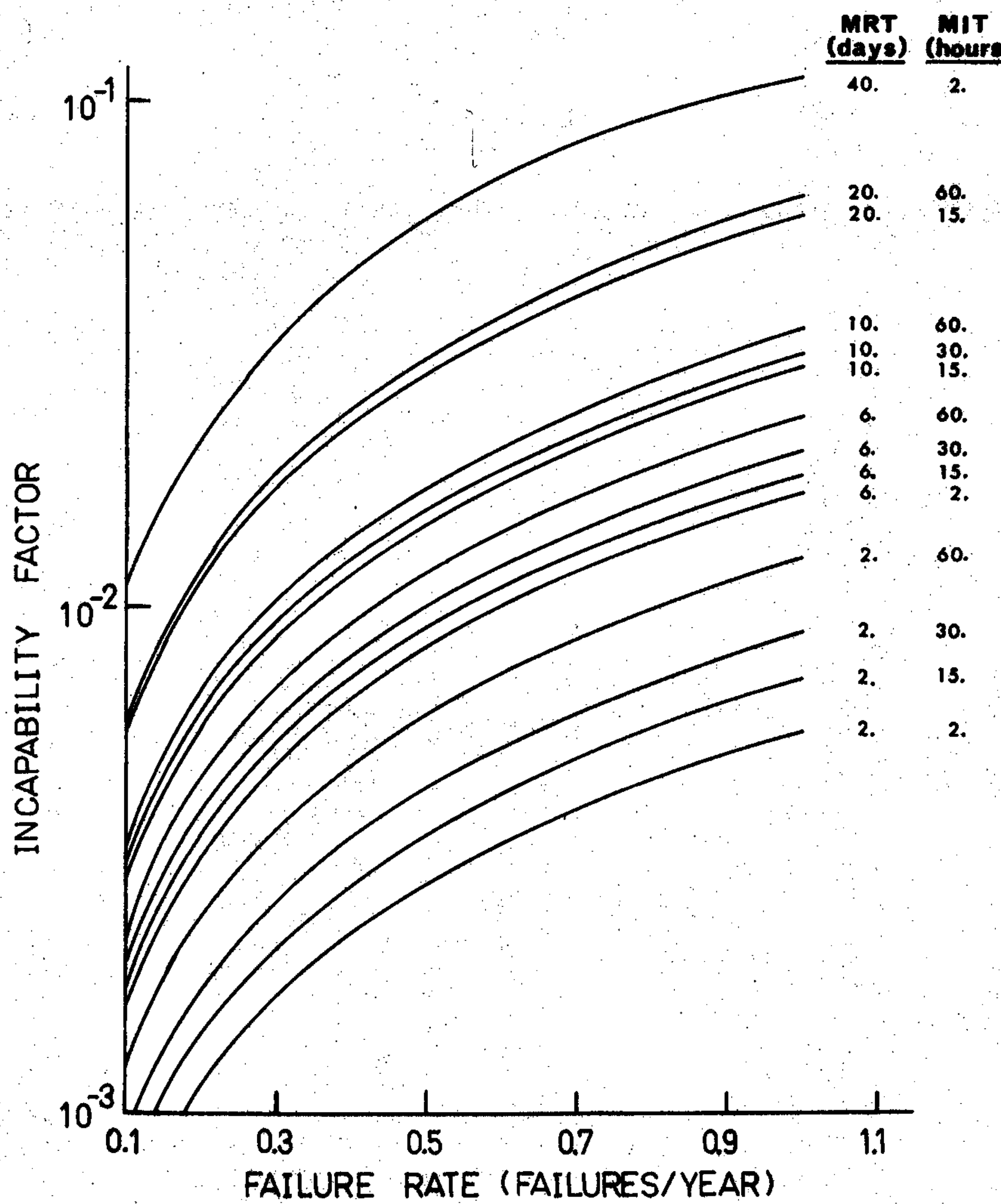


Figure 2.6: Incapability Factor as the Failure Rate is Varied.
MRT is the Mean Repair Time (MRT = $1/\mu$).
MIT is the Mean Installation Time (MIT = $1/\gamma$).

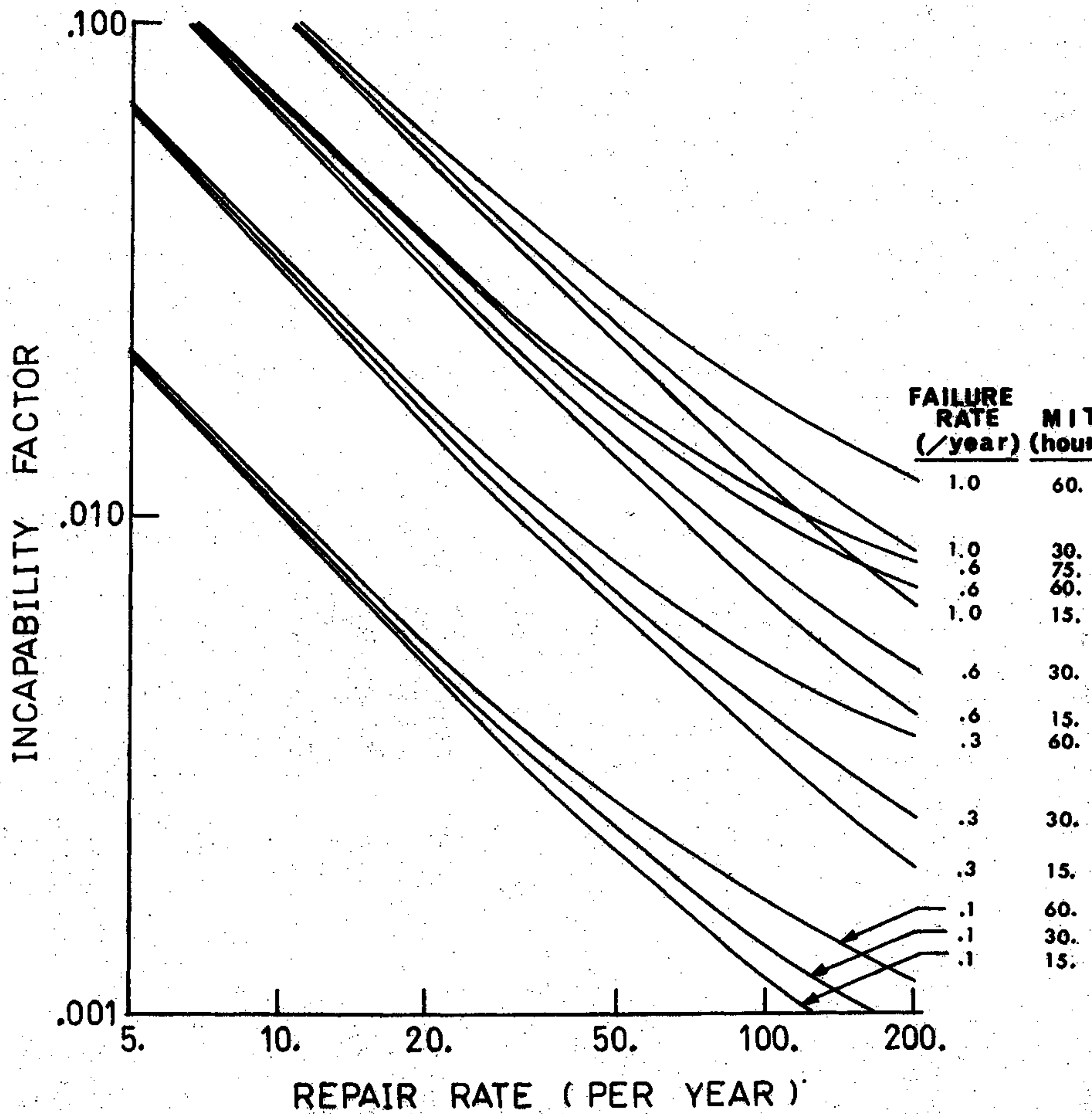


Figure 2.7: Incapability Factor as the Repair Rate is Varied. Mean Installation Time Designated as MIT is the Reciprocal of the Installation Rate.

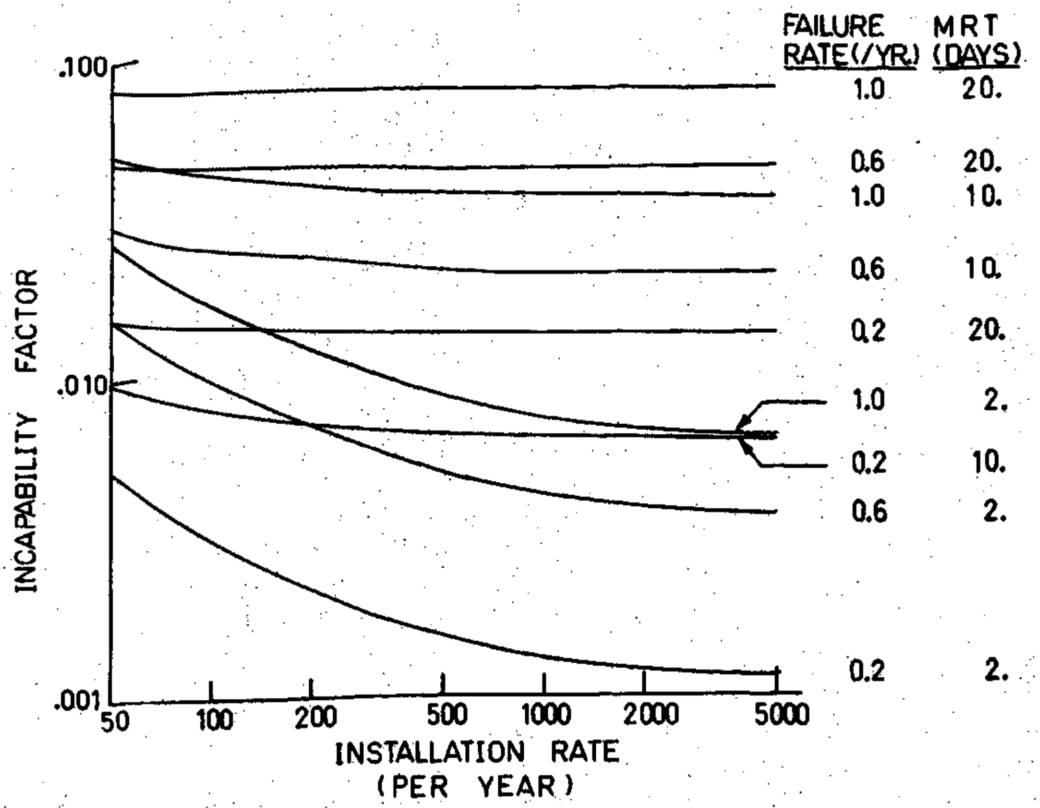


Figure 2.8: Incapability Factor as the Installation Rate is Varied. MRT is the Mean Repair Time.

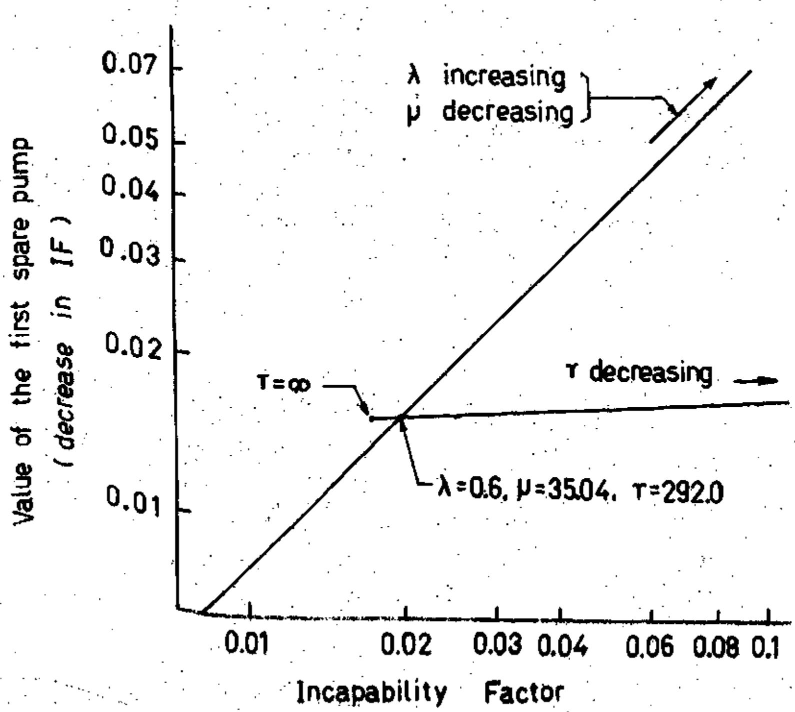


Figure 2.9: Value of the First Spare as the Transition Rates are Varied.

Table 2.8: Incapability Factor of Four Unit System for Different Cases of Repair Restriction

Repair Restricted to the Indicated Number of Pumps	Incapability Factor — Number of Spares					
	0	1	2	3	4	5
1	0.0260725	0.0085553	0.0038251	0.0025419	0.0021933	0.0020985
2	0.0200566	0.0045103	0.0023944	0.0021078	0.0020691	0.0020639
3	0.0197594	0.0042676	0.0022625	0.0020810	0.0020647	0.0020633
4	0.0197473	0.0042541	0.0022525	0.0020759	0.0020640	0.0020632
No restriction	0.0197473	0.0042536	0.0022519	0.0020755	0.0020638	0.0020631

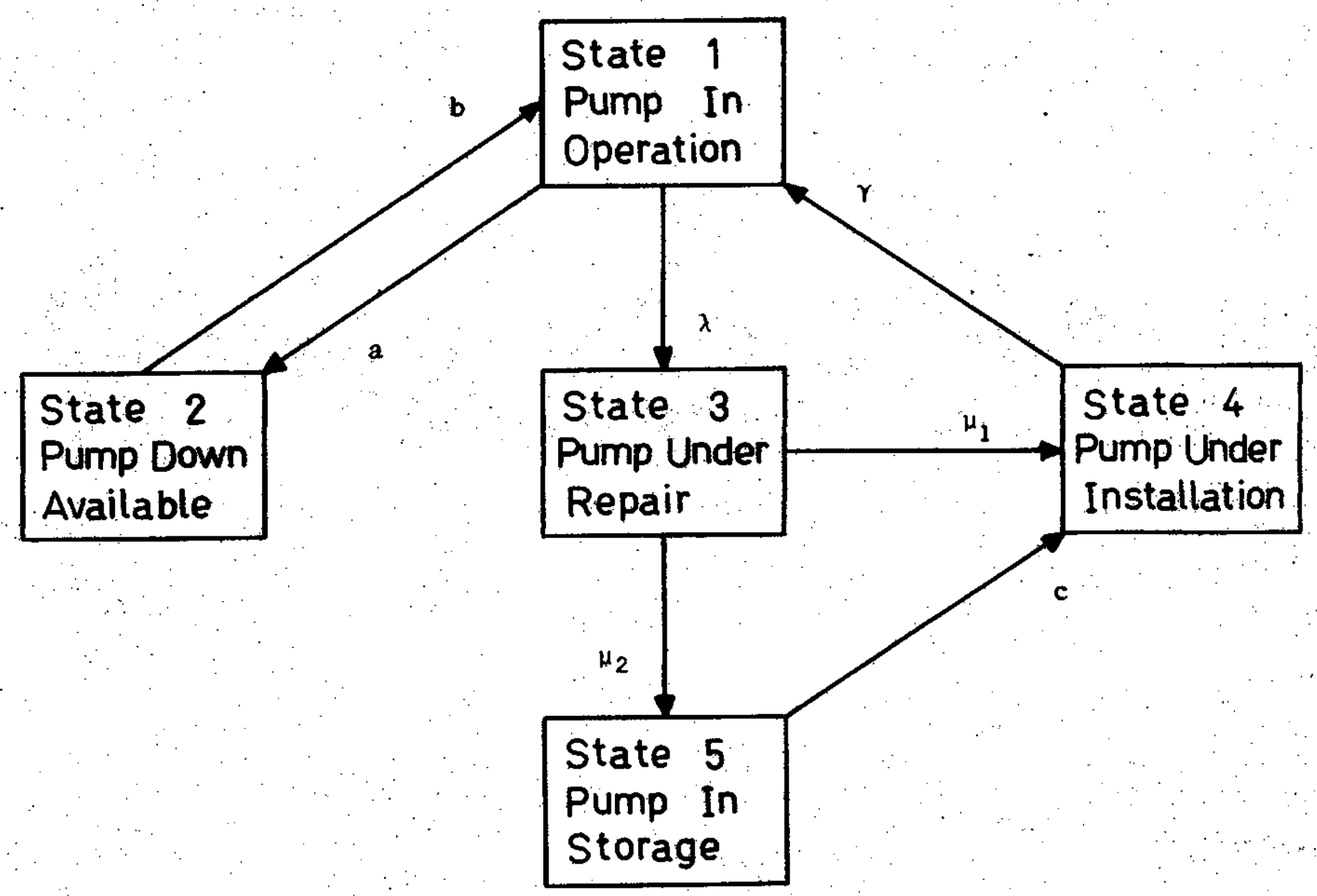


Figure 2.10: Equivalent Model for a Pump in Four Unit Operation.

are five possible states: state 1, in which the pump operates; state 2, in which it cannot operate due to other pump failures; state 3, in which it is being repaired; state 4, in which it is undergoing installation; state 5, in which it is stored in the warehouse.

The state probabilities and equivalent transition rates between the states were calculated and are shown in Table 2.9. A pump operates for 98 percent of the time for the no spare case and 79 percent of the time for the case of four spare pumps. As the number of spares increases, there are fewer complete unit shut downs resulting in less involuntary pump outages. The probability of state 2, therefore, decreases. In the no spare case there are no pumps in the warehouse and the probability of being in the state 5 is zero. As more spares are provided, the probability of pump storage in the warehouse increases. A pump will be stored in the warehouse about 18.6 percent of time for the four spare case. It will be seen in this part that the pumps must be considered collectively for accurate reliability evaluation.

2.3 The Heat Transfer Pump Models Under the Mixed Failure Modes Assumption

The pump failures are grouped into the two general failure modes designated as permanent and temporary failures. A permanent pump failure results in the pump removal from the operating site to the repair shop. Temporary failure is less serious than permanent failure as a temporarily failed pump can be repaired on the operating site in a relatively short time. Installation is not necessary after the repair. If a large number of spares are provided for a particular part, failures of the part can be considered as temporary failures because only installations are required for restoration.

Table 2.9: State Probabilities and Transition Rates of the Equivalent Model of a Pump in the Four Unit System

State Probabilities

State	SP=0	SP=1	SP=2	SP=3	SP=4
	Number of Spares				
1	0.98024991	0.93717297	0.88688718	0.84035748	0.79834898
2	0.00095078	0.00010296	0.00001835	0.00001105	0.00001008
3	0.01678510	0.01604748	0.01518642	0.01438968	0.01367036
4	0.00201421	0.00192570	0.00182237	0.00172676	0.00164044
5	0.0	0.04475089	0.09608567	0.14351502	0.18633014
	1.0	1.0	1.0	1.0	1.0

State	SP=0	SP=1	SP=2	SP=3	SP=4
up	0.98024991	0.93717297	0.88688718	0.84035748	0.79834898
down	0.01975009	0.06282703	0.11311281	0.15964251	0.20165102
	1.0	1.0	1.0	1.0	1.0

Equivalent Transition Rates

Rate	SP=0	SP=1	SP=2	SP=3	SP=4
a	6.5549×10^{-2}	1.4587×10^{-3}	7.9762×10^{-4}	7.3926×10^{-4}	7.3540×10^{-4}
b	6.7581×10^{1}	1.3278×10^{2}	3.8550×10^{2}	5.6221×10^{2}	5.8245×10^{2}
c	∞	9.5974	5.3692	3.5039	2.5703
μ_1	3.5040×10^{1}	8.2761	1.0686	9.3475×10^{-3}	6.4143×10^{-4}
μ_2	0.0	2.6764×10^{1}	3.3971×10^{1}	3.4946×10^{1}	3.5033×10^{1}

The following symbols are used to designate the transition rates:

λ = permanent failure rate of an operating pump

μ = repair rate of a permanently failed pump

γ = installation rate of a spare

λ' = temporary failure rate of an operating pump

μ' = repair rate of a temporarily failed pump.

The transition rates are again assumed to be constant with respect to time.

2.3.1 Method of State Description

A temporarily failed pump remains on the operating site. It is represented by the symbol ⊗ in state space diagrams. There are ten possible operating conditions for a unit: state 1, where all pumps are operating (represented by 00); state 2, where one pump has been removed (0X); state 3, where an installation is in progress (0I); state 4, where two pumps have been removed (XX); state 5, where one pump has been removed and an installation is in progress (IX); state 6, where two installations are in progress (II); state 7, where a pump has temporarily failed (0⊗); state 8, where one pump has failed temporarily and another permanently (⊗X); state 9, where one pump has temporarily failed and an installation is in progress (⊗I); state 10, where two pumps have temporarily failed (⊗⊗).

The repair shop state is represented by the number of permanently failed pumps undergoing repair.

2.3.2 One Unit Model

The number of states in pump models under the mixed modes assumption is large as shown in Appendix 1. A state space diagram of a single unit model without a spare is shown in Figure 2.11. The states are shown in three separate planes for systematic model construction. A simple geometrical pattern in the diagram is produced in this way. Transition lines associated with an event are parallel. States in the first plane (π_1) do not contain any temporarily failed pumps. States in the second plane (π_2) have one temporarily failed pump. States in the third plane (π_3) have two temporarily failed pumps. Transition lines of permanent failure, repair and installation are confined in the planes. Transition lines of temporary failure and repair exist only between the planes.

Steady state reliability indices for the no spare case were obtained and listed in Table 2.10. The following transition rates were used:

$$\lambda = 0.6 \text{ f/year}, \quad \mu = 35.04 \text{ r/year}, \quad \gamma = 292 \text{ i/year},$$
$$\lambda' = 0.4 \text{ f/year}, \quad \mu' = 584 \text{ r/year}.$$

The mean repair time of temporarily failed pumps is 15 hours. Comparing Table 2.10 with Table 2.3, it was found that addition of the temporary failure mode produces little changes in the state probabilities and incapability factor, but it increases the state frequencies considerably.

State space models for a unit with one, two, three, and four spares were built by the computer method explained in Appendix 2. Stochastic transitional probability matrices were constructed from these models and then the steady state probabilities and the incapability factors were

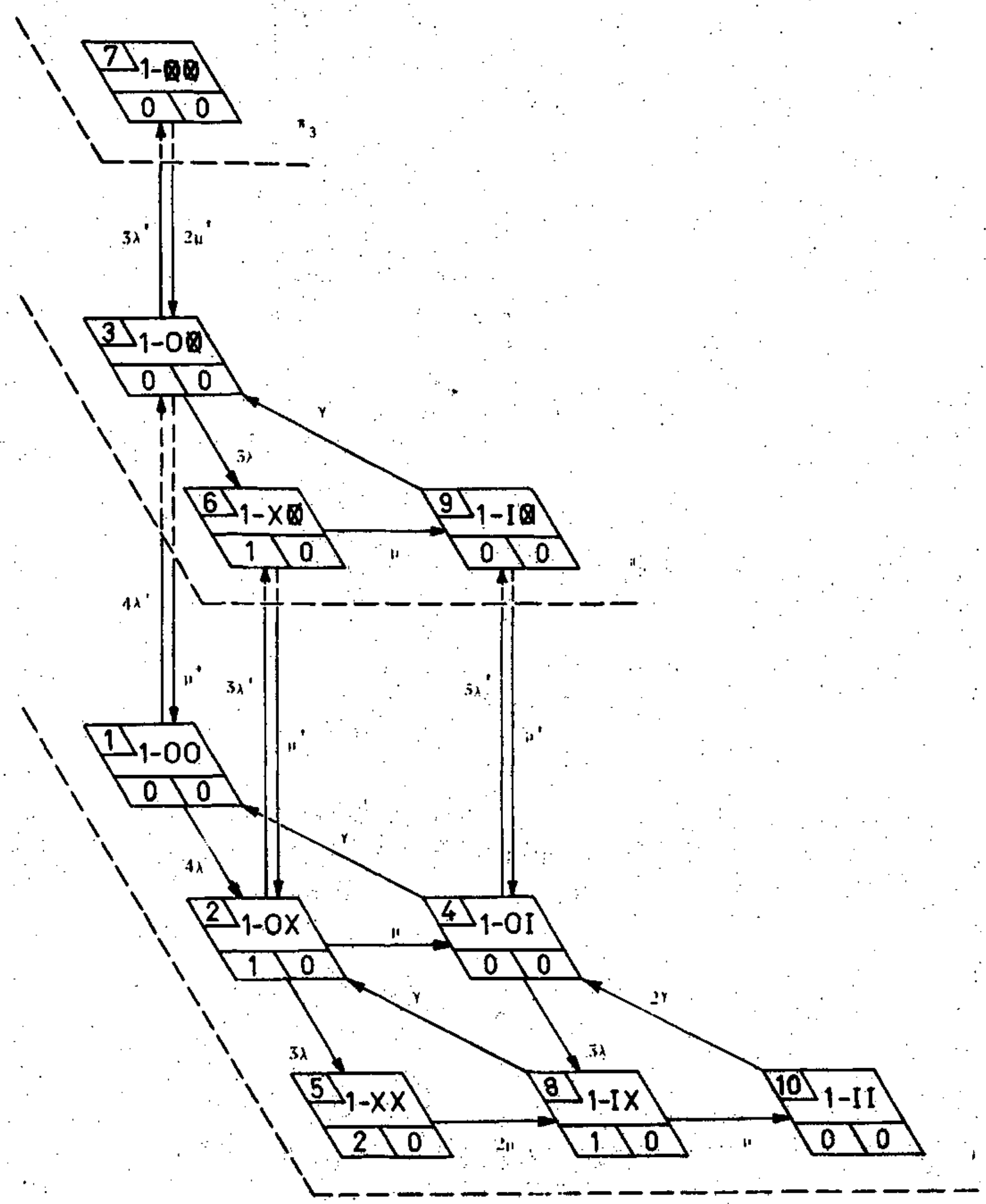


Figure 2.11: State Space Diagram of a Single Unit without Spares under the Mixed Failure Modes Assumption

Table 2.10: Steady State Reliability Indices for a Single Unit Assuming the Mixed Failure Modes

States

State No.	Availability	Frequency (per year)	Average Duration (year)
1	0.9243688	3.6974752	0.2500000
2	0.0633129	2.4084239	0.0262881
3	0.0025325	1.4865876	0.0017036
4	0.0075976	2.2412778	0.0033898
5	0.0016262	0.1139633	0.0142694
6	0.0001301	0.0805340	0.0016154
7	0.0000026	0.0030390	0.0008562
8	0.0003903	0.1276389	0.0030577
9	0.0000156	0.0136756	0.0011416
10	0.0000234	0.0136756	0.0017123

Capacity States

Capacity	Availability	Frequency (per year)	Average Duration (year)
full	0.9243688	3.6974752	0.2500000
75%	0.0734430	3.9178042	0.0187460
down	0.0021882	0.2203290	0.0099315

Incapability Factor = 0.0205488

calculated and listed in Table 2.11. The state space diagram for a unit with a spare is shown in Figure 2.12. It is also shown in three separate planes. The capacity state probabilities and incapability factor were calculated for the case of infinite spares and are listed in Table 2.11.

2.3.3 Four Units Model

The state space model for the system of four units is too large to be shown in this thesis. The state space model for two units without spare is shown in Table 2.12. It has 55 states and 194 transition rates. In building these models, it has been assumed that a pump released from the repair shop is first installed in the unit in state BX if required.

Four unit state space models have been generated truncating insignificant states and the transition matrices associated with the models were built by computer. The capacity probabilities and the incapability factor were evaluated and are listed in Table 2.13 as the number of system spares was varied from 0 to 4. The capacity state probabilities for the infinite spare case were obtained from the capacity state probability values for the single unit with an infinite number of spares shown in Table 2.11. The incapability factor for the infinite spares case must remain constant regardless of the number of units in the system. These values are listed in Table 2.13.

Table 2.11: Capacity State Probabilities and Incapability Factors Of a Unit Assuming the Mixed Failure Modes

Capacity	Probability					
	Spare 0	Spare 1	Spare 2	Spare 3	Spare 4	Spare Infinite
Full	0.9243688	0.9869959	0.9890710	0.9891151	0.9891158	0.9891158
75%	0.0734430	0.0128903	0.0108827	0.0108403	0.0108396	0.0108396
Down	0.0021882	0.0001138	0.0000464	0.0000446	0.0000445	0.0000445
	1.0	1.0	1.0	1.0	1.0	1.0
Incapability Factor	0.0205488	0.0033364	0.0027672	0.0027548	0.0027545	0.0027545

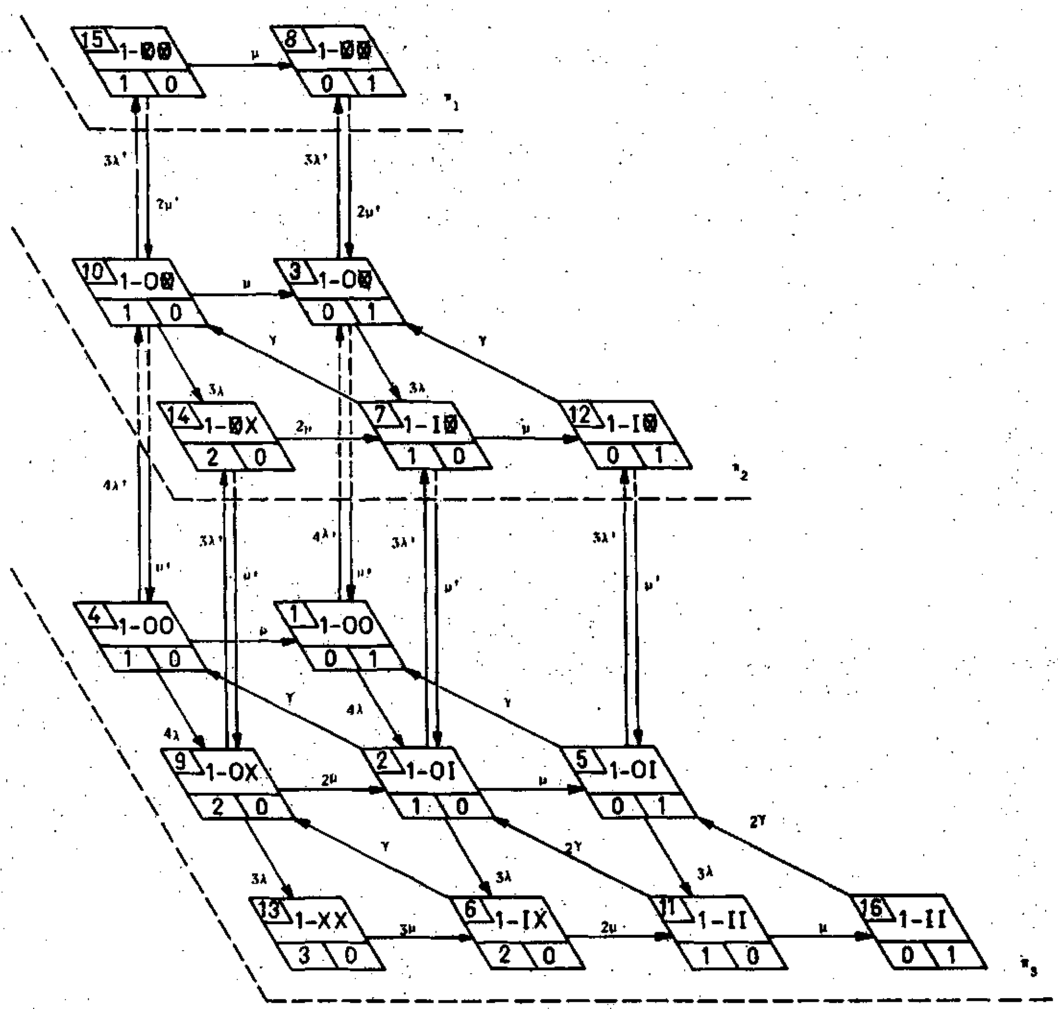


Figure 2.12: State Space Diagram of a Single Unit with a Spare under the Mixed Failure Modes Assumption

Table 2.12: State Space Model for Two Units Without Spares Assuming the Mixed Failure Modes

States

State Parameters

State No.	00	0X	0I	XX	IX	II	0Ø	ØI	ØX	ØØ	Repair shop	Ware-house
1	2	0	0	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0	1	0
3	1	0	0	0	0	0	1	0	0	0	0	0
4	1	0	1	0	0	0	0	0	0	0	0	0
5	0	2	0	0	0	0	0	0	0	0	2	0
6	1	0	0	1	0	0	0	0	0	0	2	0
7	0	1	0	0	0	0	1	0	0	0	1	0
8	1	0	0	0	0	0	0	0	1	0	1	0
9	0	0	0	0	0	0	0	0	2	0	0	0
10	1	0	0	0	0	0	0	0	0	1	0	0
11	0	1	1	0	0	0	0	0	0	0	1	0
12	1	0	0	0	1	0	0	0	0	0	1	0
13	0	0	1	0	0	0	1	0	0	0	0	0
14	1	0	0	0	0	0	0	1	0	0	0	0
15	0	1	0	1	0	0	0	0	0	0	3	0
16	0	1	0	0	0	0	0	0	1	0	2	0
17	0	0	0	1	0	0	1	0	0	0	2	0
18	0	0	0	0	0	0	1	0	1	0	1	0
19	0	1	0	0	0	0	0	0	0	1	1	0
20	0	0	0	0	0	0	1	0	0	1	0	0
21	0	0	2	0	0	0	0	0	0	0	0	0
22	0	0	1	1	0	0	0	0	0	0	2	0
23	0	1	0	0	1	0	0	0	0	0	2	0
24	0	0	1	0	0	0	0	0	1	0	1	0
25	0	1	0	0	0	0	0	1	0	0	1	0
26	1	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	1	0	1	0	0	0	1	0
28	0	0	0	0	0	0	1	1	0	0	0	0
29	0	0	1	0	0	0	0	0	0	1	0	0
30	0	0	0	2	0	0	0	0	0	0	4	0
31	0	0	0	1	0	0	0	0	1	0	3	0
32	0	0	0	0	0	0	0	0	2	0	2	0
33	0	0	0	1	0	0	0	0	0	1	2	0
34	0	0	0	0	0	0	0	0	1	1	1	0
35	0	0	0	0	0	0	0	0	0	2	0	0
36	0	0	1	0	1	0	0	0	0	0	1	0
37	0	0	1	0	0	0	0	1	0	0	0	0

Table 2.12 (Continued)

State Parameters

State No.	00	0X	0I	XX	IX	II	0●	●I	●X	●●	Repair shop	Ware-house
38	0	0	0	1	1	0	0	0	0	0	3	0
39	0	0	0	1	0	0	0	1	0	0	2	0
40	0	1	0	0	0	1	0	0	0	0	1	0
41	0	0	0	0	1	0	0	0	1	0	2	0
42	0	0	0	0	0	0	0	1	1	0	1	0
43	0	0	0	0	0	1	1	0	0	0	0	0
44	0	0	0	0	1	0	0	0	0	1	1	0
45	0	0	0	0	0	0	0	1	0	1	0	0
46	0	0	1	0	0	1	0	0	0	0	0	0
47	0	0	0	0	2	0	0	0	0	0	2	0
48	0	0	0	0	1	0	0	1	0	0	1	0
49	0	0	0	0	0	0	0	2	0	0	0	0
50	0	0	0	1	0	1	0	0	0	0	2	0
51	0	0	0	0	0	1	0	0	1	0	1	0
52	0	0	0	0	0	1	0	0	0	1	0	0
53	0	0	0	0	1	1	0	0	0	0	1	0
54	0	0	0	0	0	1	0	1	0	0	0	0
55	0	0	0	0	0	2	0	0	0	0	0	0

Transition Rates

From	To	Rate	From	To	Rate	From	To	Rate
1	2	8λ	1	3	$8\lambda'$	2	4	μ
2	5	4λ	2	6	3λ	2	7	$4\lambda'$
2	8	$3\lambda'$	3	1	μ'	3	7	4λ
3	8	3λ	3	9	$4\lambda'$	3	10	$3\lambda'$
4	1	γ	4	11	4λ	4	12	3λ
4	13	$4\lambda'$	3	14	$3\lambda'$	5	11	2μ
5	15	6λ	5	16	$6\lambda'$	6	12	2μ
6	15	4λ	6	17	$4\lambda'$	7	13	μ
7	2	μ'	7	17	3λ	7	16	3λ
7	18	$3\lambda'$	7	19	$3\lambda'$	8	14	μ
8	2	μ'	8	16	4λ	8	18	$4\lambda'$
9	3	$2\mu'$	9	18	6λ	9	20	$6\lambda'$
10	3	2μ	10	19	4λ	10	20	$4\lambda'$
11	2	γ	11	21	μ	11	22	3λ
11	23	3λ	11	24	$3\lambda'$	11	25	$3\lambda'$
12	2	γ	12	26	μ	12	23	4λ
12	27	$4\lambda'$	13	3	γ	13	4	μ'
13	27	3λ	13	24	3λ	13	28	$3\lambda'$
13	29	$3\lambda'$	14	3	γ	14	4	μ'
14	25	4λ	14	28	$4\lambda'$	15	23	3μ
15	30	3λ	15	31	$3\lambda'$	16	25	2μ

Table 2.12 (Continued)

From	To	Rate	From	To	Rate	From	To	Rate
16	5	μ'	16	31	3λ	16	32	$3\lambda'$
17	27	2μ	17	6	μ'	17	31	3λ
17	33	$3\lambda'$	18	28	μ	18	8	μ'
18	7	μ'	18	32	3λ	18	34	$3\lambda'$
19	29	μ	19	7	$2\mu'$	19	33	3λ
19	34	$3\lambda'$	20	10	μ	20	9	$2\mu'$
20	33	3λ	20	35	$3\lambda'$	21	4	2γ
21	36	6λ	21	37	$6\lambda'$	22	6	1γ
22	36	2μ	22	38	3λ	22	39	$3\lambda'$
23	5	γ	23	40	2μ	23	38	3λ
23	41	$3\lambda'$	24	8	γ	24	37	μ
24	11	μ'	24	41	3λ	24	42	$3\lambda'$
25	7	γ	25	37	μ	25	11	μ'
25	39	3λ	25	42	$3\lambda'$	26	4	2γ
26	40	4λ	26	43	$4\lambda'$	27	7	γ
27	43	μ	27	12	μ'	27	41	3λ
27	44	$3\lambda'$	28	9	γ	28	14	μ'
28	13	μ'	28	42	3λ	28	45	$3\lambda'$
29	10	γ	29	13	$2\mu'$	29	44	3λ
29	45	$3\lambda'$	30	38	4μ	31	41	3μ
31	15	μ'	32	42	2μ	32	16	$2\mu'$
33	44	2μ	33	17	$2\mu'$	34	45	μ
34	19	μ'	34	18	$2\mu'$	35	20	$4\mu'$
36	12	γ	36	11	γ	36	46	μ
36	47	3λ	36	48	$3\lambda'$	37	14	γ
37	13	γ	37	21	μ'	37	48	3λ
37	49	$3\lambda'$	38	15	γ	34	47	3μ
39	17	γ	39	48	2μ	39	22	μ'
40	11	2γ	40	46	μ	40	50	3λ
40	51	$3\lambda'$	41	16	γ	41	51	2μ
41	23	μ'	42	18	γ	42	49	μ
42	24	μ'	42	25	μ'	43	13	2γ
43	26	μ'	43	51	3λ	43	52	$3\lambda'$
44	19	γ	44	52	μ	45	27	$2\mu'$
45	20	γ	45	29	μ'	45	28	$2\mu'$
46	26	γ	46	21	2γ	46	53	3λ
46	54	$3\lambda'$	47	23	2γ	47	53	2μ
48	25	γ	48	27	γ	48	54	μ
48	36	μ'	49	28	2γ	49	37	$2\mu'$
50	22	2γ	50	53	2μ	51	24	2γ
51	54	μ	51	40	μ'	52	29	2γ
52	43	$2\mu'$	53	40	γ	53	36	2γ
53	55	μ	54	37	2γ	54	43	γ
54	46	μ'	55	46	4γ			

Table 2.13: Capacity State Probabilities and Incapability Factors For Four Unit Operation Assuming the Mixed Failure Modes

Number of Pumps in Operation	Probability					
	Spare 0	Spare 1	Spare 2	Spare 3	Spare 4	Spare ∞
16	0.7300980	0.9279224	0.9546342	0.9570050	0.9571608	0.9571690
15	0.2320311	0.0673013	0.0441672	0.0421000	0.0419653	0.0419581
14	0.0281475	0.0036321	0.0009374	0.0007066	0.0006907	0.0006897
13	0.0015444	0.0001287	0.0000138	0.0000056	0.0000050	0.0000050
12	0.0064492	0.0008693	0.0002320	0.0001765	0.0001727	0.0001724
11	0.0015749	0.0001361	0.0000154	0.0000063	0.0000056	0.0000057
10	0.0001316	0.0000085	0.0	0.0	0.0	0.0
8	0.0000234	0.0000015	0.0	0.0	0.0	0.0
	1.0	1.0	1.0	1.0	1.0	1.0
I.F.	0.0204755	0.0049483	0.0029430	0.0027667	0.0027550	0.0027545

2.4 The Heat Transport Pump Models Under The Two Failure Modes Assumption

The pump failures are now grouped into two permanent failure modes: one requiring 100 hours of average repair time; the other requiring 1000 hours of average repair time. Temporary pump failures are ignored as in Section 2.2. Each permanent failure mode is represented by a constant failure rate. The following symbols are used to designate the transition rates.

λ_1 = failure rate of a pump requiring 100 hours of average repair time (= 0.5 f/year).

λ_2 = failure rate of a pump requiring 1000 hours of average repair time (=0.1 f/year).

μ_1 = 1 repair per 100 hours.

μ_2 = 1 repair per 1000 hours.

γ = installation rates (= 292.0 /year).

The repair process is now represented by two parallel exponential distributions.

2.4.1 State Description Method

The method of state representation is identical with the method explained in Section 2.2.1 except that two numbers are needed to describe the repair shop: the number of failed pumps undergoing repair process 1; and the number of failed pumps undergoing repair process 2.

2.4.2 Single Unit Model

The state space diagram for the case of no spare is shown in Figure 2.13. The states are shown in three planes. The state probabilities

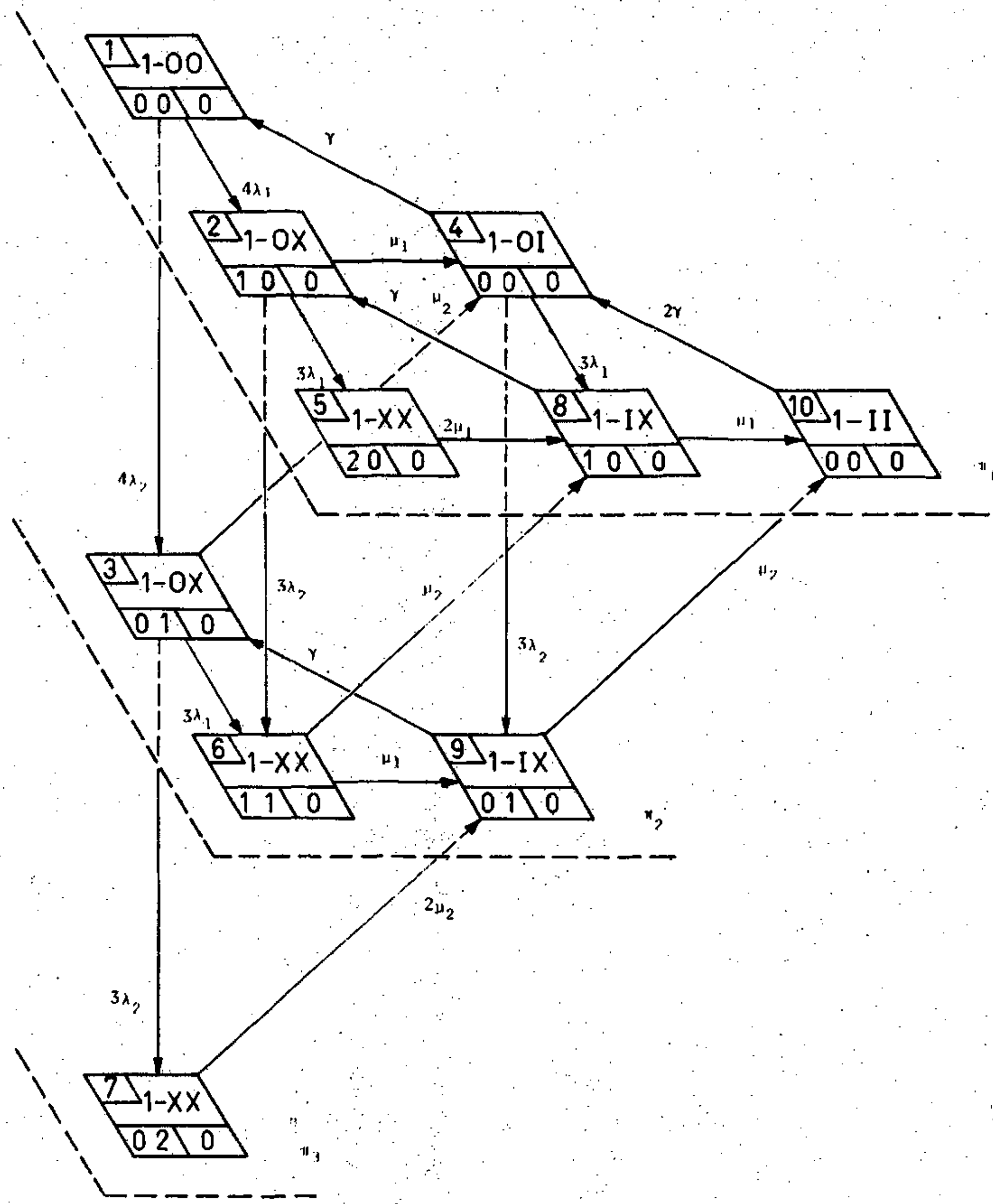


Figure 2.13: State Space Diagram of a Single Unit Without Spares under the Two Permanent Failure Modes Assumption

and frequencies were obtained and listed in Table 2.14. The probabilities and frequencies for different capacity states are identical with those shown in Table 2.3. Figure 2.14 shows the state space diagram for the one spare pump case. The capacity state probabilities and incapability factor were calculated for cases with different number of spares and are listed in Table 2.15. Those quantities for the infinite spare case are also shown in the table. Table 2.15 can be compared with Table 2.3 to observe the effect of assuming two permanent failure modes instead of one. The effect is very small.

2.4.3 Four Unit Model

The state space models for the four unit system were constructed using the computer method presented in Appendix 2. The number of states is extremely large for a four unit model as shown in Appendix 1, and requires a considerable number of transitions. A complete two unit model with one spare which is shared between the two units is shown in Table 2.16.

State probabilities were calculated for the four unit case in which the number of spares were 0, 1, 2, 3, and 4. The capacity state probabilities are listed in Table 2.17, which also includes the results for the infinite spare case. The probabilities for infinite spares are the same as those obtained assuming a single failure mode. The effect of four spares on the four unit system is virtually identical to that obtained assuming an infinite number of spares. In this case there is very little benefit in providing more than four spares.

Table 2.14: Steady State Reliability Indices for a Unit Without a Spare Assuming the Two Failure Modes

States

State No.	Probability	Frequency (per year)	Average Duration (year)
1	0.9268535	2.2244485	0.4166667
2	0.0211610	1.8917969	0.0111857
3	0.0423221	0.4469212	0.0946970
4	0.0001812	0.0317416	0.0057078
5	0.0007247	0.0698314	0.0103778
6	0.0076180	2.2381608	0.0034037
7	0.0007247	0.0126966	0.0570776
8	0.0001304	0.0495168	0.0026344
9	0.0002609	0.0784651	0.0033249
10	0.0000235	0.0137124	0.0017123

Capacity States

Capacity	Probability	Frequency (per year)	Average Duration (year)
Full	0.9268535	2.2244485	0.4166667
75%	0.0711011	2.3524304	0.0302245
Down	0.0020454	0.1279820	0.0159817

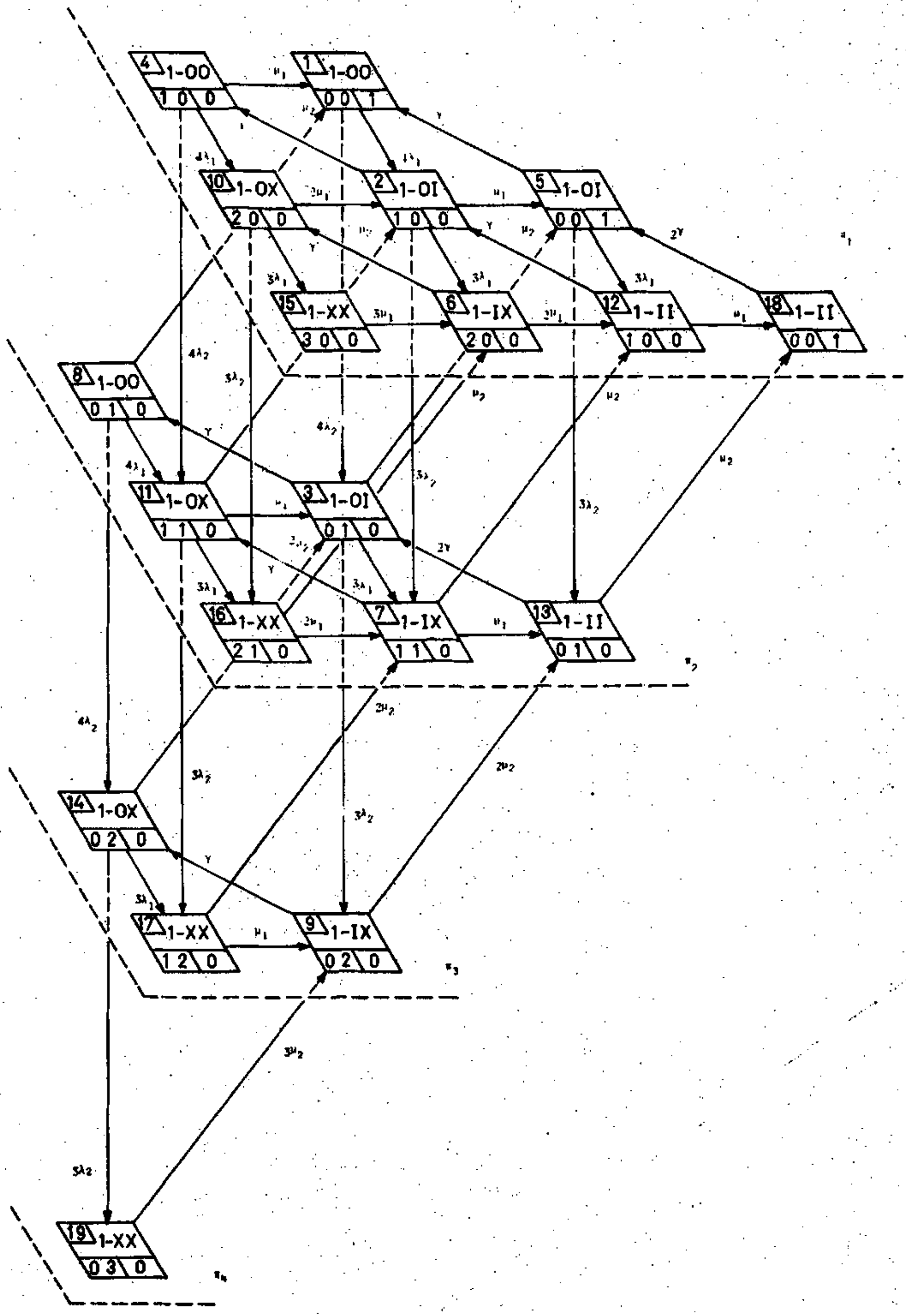


Figure 2.14: State Space Diagram of a Single Unit with a Spare Under the Two Permanent Failure Modes Assumption.

Table 2.15: Capacity State Probabilities and Incapability Factors Of a Unit Assuming the Two Failure Modes

	Probability				
Capacity	0 Spare	1 Spare	2 Spares	3 Spares	Infinite Spares
100%	0.9268535	0.9896660	0.9917759	0.9918221	0.9918230
75%	0.0711011	0.0102468	0.0081974	0.0081527	0.0081519
0%	0.0020454	0.0000872	0.0000267	0.0000252	0.0000251
	1.0	1.0	1.0	1.0	1.0
Incapability Factor	0.0198207	0.0026489	0.0020761	0.0020634	0.0020632

Table 2.16: State Space Model of Two Units With One Common Spare Assuming the Two Failure Modes

	States								
	Operating Site						Repair Shop		
State Number	00	0X	0I	XX	IX	II	Mode 1	Mode 2	Warehouse
1	2	0	0	0	0	0	0	0	1
2	1	0	1	0	0	0	1	0	0
3	1	0	1	0	0	0	0	1	0
4	0	1	1	0	0	0	2	0	0
5	1	0	0	0	1	0	2	0	0
6	0	1	1	0	0	0	1	1	0
7	1	0	0	0	1	0	1	1	0
8	1	0	1	0	0	0	0	0	1
9	2	0	0	0	0	0	1	0	0
10	0	1	1	0	0	0	0	2	0
11	1	0	0	0	1	0	0	2	0
12	2	0	0	0	0	0	0	1	0
13	0	0	1	1	0	0	3	0	0
14	0	1	0	0	1	0	3	0	0
15	0	0	1	1	0	0	2	1	0
16	0	1	0	0	1	0	2	1	0
17	0	0	2	0	0	0	1	0	0
18	1	1	0	0	0	0	2	0	0
19	1	0	0	0	0	1	1	0	0
20	0	0	1	1	0	0	1	2	0
21	0	1	0	0	1	0	1	2	0
22	0	0	2	0	0	0	0	1	0
23	1	1	0	0	0	0	1	1	0
24	1	0	0	0	0	1	0	1	0
25	0	0	1	1	0	0	0	3	0
26	0	1	0	0	1	0	0	3	0
27	1	1	0	0	0	0	0	2	0
28	0	0	0	1	1	0	4	0	0
29	0	0	0	1	1	0	3	1	0
30	0	0	1	0	1	0	2	0	0
31	1	0	0	1	0	0	3	0	0
32	0	1	0	0	0	1	2	0	0
33	0	2	0	0	0	0	3	0	0
34	0	0	0	1	1	0	2	2	0
35	0	0	1	0	1	0	1	1	0
36	1	0	0	1	0	0	2	1	0
37	0	1	0	0	0	1	1	1	0
38	0	2	0	0	0	0	2	1	0
39	0	0	2	0	0	0	0	0	1

Table 2.16 (Continued)

State Number	Operating Site 00	0X	0I	XX	IX	II	Repair Shop Mode 1	Mode 2	Warehouse
40	1	0	0	0	0	1	0	0	1
41	0	0	0	1	1	0	1	3	0
42	0	0	1	0	1	0	0	2	0
43	1	0	0	1	0	0	1	2	0
44	0	1	0	0	0	1	0	2	0
45	0	2	0	0	0	0	1	2	0
46	0	0	0	1	1	0	0	4	0
47	1	0	0	1	0	0	0	3	0
48	0	2	0	0	0	0	0	3	0
49	0	0	0	0	2	0	3	0	0
50	0	1	0	1	0	0	4	0	0
51	0	0	0	0	2	0	2	1	0
52	0	1	0	1	0	0	3	1	0
53	0	0	1	0	0	1	1	0	0
54	0	0	0	1	0	1	3	0	0
55	0	0	0	1	0	1	2	1	0
56	0	0	0	0	2	0	1	2	0
57	0	1	0	1	0	0	2	2	0
58	0	0	1	0	0	1	0	1	0
59	0	0	0	1	0	1	1	2	0
60	0	0	0	0	2	0	0	3	0
61	0	1	0	1	0	0	1	3	0
62	0	0	0	1	0	1	0	3	0
63	0	1	0	1	0	0	0	4	0
64	0	0	0	0	1	1	2	0	0
65	0	0	0	2	0	0	5	0	0
66	0	0	0	2	0	0	4	1	0
67	0	0	0	0	1	1	1	1	0
68	0	0	0	2	0	0	3	2	0
69	0	0	1	0	0	1	0	0	1
70	0	0	0	0	1	1	0	2	0
71	0	0	0	2	0	0	2	3	0
72	0	0	0	2	0	0	1	4	0
73	0	0	0	2	0	0	0	5	0
74	0	0	0	0	0	2	1	0	0
75	0	0	0	0	0	2	0	1	0
76	0	0	0	0	0	2	0	0	1

Transition Rates

From	To	Rate	From	To	Rate	From	To	Rate
1	2	4.0	6	21	0.3	12	23	4.0
1	3	.8	6	22	87.6	12	27	0.8
2	4	2.0	6	17	8.76	12	1	8.76

Table 2.16 (Continued)

From	To	Rate	From	To	Rate	From	To	Rate
2	5	1.5	6	23	292	13	38	1.5
2	6	0.4	7	16	2.0	13	29	0.3
2	7	0.3	7	21	0.4	13	30	262.8
2	8	87.6	7	24	87.6	13	31	292.0
2	9	292.0	7	19	8.76	14	28	1.5
3	6	2.0	7	23	292.0	14	29	0.3
3	7	1.5	8	17	2.0	14	32	262.8
3	10	0.4	8	19	1.5	14	33	292.0
3	11	0.3	8	22	0.4	15	29	1.5
3	8	8.76	8	24	0.3	15	34	0.3
3	12	292	8	1	292.0	15	35	175.2
4	13	1.5	9	18	4.0	15	30	8.76
4	14	1.5	9	23	0.8	15	36	292
4	15	0.3	9	1	87.6	16	29	1.5
4	16	0.3	10	20	1.5	16	34	0.3
4	17	175.2	10	21	1.5	16	37	175.2
4	18	292	10	25	0.3	16	32	8.76
5	14	2.0	10	26	0.3	16	38	292
5	16	0.4	10	22	17.52	17	30	3.0
5	19	175.2	10	27	292	17	35	0.6
5	18	292.0	11	21	2.0	17	39	87.6
6	15	1.5	11	26	0.4	17	2	584.0
6	16	1.5	11	24	17.52	18	33	2.0
6	20	0.3	11	27	292	18	31	1.5
18	38	0.4	27	45	2.0	37	59	0.3
18	36	0.3	27	43	1.5	37	58	87.6
18	2	175.2	27	48	0.4	37	53	8.76
19	32	2.0	27	47	0.3	37	6	584.0
19	37	0.4	27	3	17.52	38	52	3.0
19	40	87.6	28	49	350.4	38	57	0.6
19	2	584.0	28	50	292.0	38	6	175.2
20	34	1.5	29	51	262.8	38	4	8.76
20	41	0.3	29	49	8.76	39	53	3.0
20	42	87.6	29	52	292.0	39	58	0.6
20	35	17.52	30	49	1.5	39	8	584.0
20	43	292.0	30	51	0.3	40	53	2.0
21	34	1.5	30	53	175.2	40	58	0.4
21	41	0.3	30	5	292.0	40	8	584.0
21	44	87.6	30	4	292.0	41	60	87.6
21	37	17.52	31	50	2.0	41	56	26.28
21	45	292.0	31	52	0.4	41	61	292.0
22	35	3.0	21	5	262.8	42	56	1.5
22	42	0.6	32	54	1.5	42	60	0.3
22	39	8.76	32	55	0.3	42	58	17.52
22	3	584.0	32	53	175.2	42	11	292.0
23	38	2.0	32	4	584.0	42	10	292.0
23	36	1.5	33	50	3.0	43	57	2.0
23	45	0.4	33	52	0.6	43	61	0.4
23	43	0.3	33	4	262.8	43	11	87.6

Table 2.16 (Continued)

From	To	Rate
23	3	87.6
23	2	8.76
24	37	2.0
24	44	0.4
24	40	8.76
24	3	584.0
25	41	1.5
25	46	0.3
25	42	26.28
25	47	292.0
26	41	1.5
26	46	0.3
26	44	26.28
26	48	292.0
48	61	3.0
48	63	0.6
48	10	26.28
49	64	262.8
49	14	584
50	65	1.5
50	66	0.3
50	14	350.4
51	67	175.2
51	64	8.76
51	16	584.0
52	66	1.5
52	68	0.3
52	16	262.8
52	14	8.76
53	64	1.5
53	67	0.3
53	69	87.6
53	19	292
53	17	584.0
54	64	262.8
54	13	584.0
55	67	175.2
55	64	8.76
55	15	584.0
56	70	87.6
56	67	17.52
56	21	584.0
57	68	1.5
57	71	0.3
57	21	175.2
57	16	17.52
58	67	1.5
58	70	0.3
58	69	8.76
58	24	292.0
58	22	584.0
59	70	87.6

From	To	Rate
34	56	175.2
34	51	17.52
34	57	292.0
35	51	1.5
35	56	0.3
35	58	87.6
35	53	8.76
35	7	292.0
35	6	292.0
36	52	2.0
36	57	0.4
36	7	175.2
26	5	8.76
37	55	1.5
59	67	17.52
59	20	584.0
60	70	26.28
60	26	584.0
61	71	1.5
61	72	0.3
61	26	87.6
61	21	26.28
62	70	26.28
62	25	584.0
63	72	1.5
63	73	0.3
63	26	35.04
64	74	175.2
64	32	292.0
64	30	584.0
65	28	438.0
66	29	350.4
66	28	8.76
67	75	87.6
67	74	8.76
67	37	292.0
67	35	584.0
68	34	262.8
68	29	17.52
69	74	1.5
69	75	0.3
69	40	292.0
69	39	584.0
70	75	17.52
70	44	292.0
70	42	584.0
71	41	175.2
71	34	26.28
72	46	87.6
72	41	35.04
73	46	43.8
74	76	87.6

From	To	Rate
43	7	17.52
44	59	1.5
44	62	0.3
44	58	17.52
44	10	584.0
45	57	3.0
45	61	0.6
45	10	87.6
45	6	17.52
46	60	35.04
46	63	292.0
47	61	2.0
47	65	0.4
47	11	26.28
74	53	1168.0
75	76	8.76
75	78	1168.0
76	69	1168.0

Table 2.17: Capacity State Probabilities and Incapability Factors For Four Unit Operation Assuming the Two Failure Modes

	Probability					
No. on Pumps in Operation	Spare 0	Spare 1	Spare 2	Spare 3	Spare 4	Infinite Spares
16	0.7379677	0.9379603	0.9652570	0.9675197	0.9676821	0.9676906
15	0.2264416	0.0579445	0.0341865	0.0319655	0.0318225	0.0318145
14	0.0269285	0.0031374	0.0004366	0.0004074	0.0003926	0.0003922
13	0.0014743	0.0001130	0.0000052	0.0000026	0.0000022	0.0000021
12	0.0056737	0.0007228	0.0001089	0.0001017	0.0000982	0.0000981
11	0.0013725	0.0001140	0.0000058	0.0000030	0.0000025	0.0000024
10	0.0001162	0.0000067	0.0	0.0	0.0	0.0
9	0.0000036	0.0	0.0	-	-	-
8	0.0000184	0.0000012	-	-	-	-
7	0.0000034	0.0	-	-	-	-
	1.0	1.0	1.0	1.0	1.0	1.0
Incapability Factor	0.0196987	0.0042543	0.0022212	0.0020756	0.0020637	0.0020631

2.5 Conclusion

This chapter has presented a series of possible reliability models for a heat transport pump configuration similar to that used in the Bruce nuclear power plant. It should be appreciated that the results obtained are valid only under the stated assumptions. The reliability model is simplest under the single permanent failure mode assumption. It is much more complicated under the other assumptions. These assumptions, however, are not necessary if the system availability is the only important reliability index, because the availabilities for the three different cases are approximately the same. Temporary failures differ from the permanent failures not only in the repair time duration but also in the number of steps required for the failed pump restoration. If temporary failures are included in the system analysis, the frequency index shows significant change. Temporary failures cannot be ignored if the frequency is an important reliability index.

The importance of the frequency depends on the cost associated with the occurrence of a failure in addition to the loss of generation of energy which can be measured by the availability index. A pump failure may result in a long and costly plant shut-down and start-up situation. The expense of pump repair and installation is directly related to the frequency of pump failure. Provision of spares improve the system availability, not the frequency. The improvement in energy production by spares is achieved at the expense of the extra pump costs, more frequent pump repairs and installations and more room for spare pump storage.

The development of a valid model for complicated systems, such as those considered in this chapter, requires the following steps:

(1) definition of the problem and the applicable assumptions.

(2) establishment of a suitable method of state representation.

(3) state space model construction with or without truncation.

(4) evaluation of steady state reliability indices.

It is difficult to construct a large model. As illustrated in this chapter, it is possible to use a digital computer to construct large models. It is often necessary to truncate the complete model prior to solution. It is sometimes more practical to generate only the significant states instead of constructing the complete model before reducing it. This method was utilized in the case of four units. The number of states must be known before the model is constructed by the computer method because the number of states determines the size of the required memory space for the computer program.

3. RELIABILITY MODELS USING THE METHOD OF STAGES

3.1 Introduction

Work has been done (references 7, 8 and 9) in recent years on the analysis of power system reliability models involving non-exponential time distributions. Exponential distributions are often used to model the times between component failures. The down or repair times cannot always be adequately represented by this distribution. Some authors have proposed Weibull and Lognormal as possible representations. Even if the distribution for the down time were known, the analysis involving non-exponential distributions is difficult as the associated stochastic process becomes non-Markovian. There are three basic methods for reliability modelling in systems with non-Markovian characteristics: the method of supplementary variables, semi-Markov processes and the device of stages. A comparative study of these methods is done in reference (7) by application to a relatively simple model.

The device of stages has the advantage that once a proper stage combination has been found to approximate a distribution, the numerical solution is obtained in a routine manner even with fairly complex models. The other techniques become complicated and often impractical when applied to realistic problems. Some fundamental stage combinations which can be used to approximate a variety of hazard rate functions are presented in this chapter. These combinations can be used either to approximate a known distribution or to fit the given data. The parameters for the stages are obtained using moment matching. The method of stages is applied to a simple generation system model and the non-exponential down time is approximated by stage combinations.

3.2 Method of Stages

When all the random variables are exponentially distributed, the associated stochastic process is a time homogeneous Markov process. The interstate transition rates are constant and the state probabilities and frequencies can be obtained in a relatively straight-forward manner. Most of the work in power system reliability is based on essentially Markovian models. When, however, some of the random variables such as the repair times have non-exponential distributions, the inter-state transition rates become functions of the time spent in the states and the process becomes non-Markovian. The solution of non-Markovian models is, in general, difficult and the analytical techniques are usually of limited application in practical problems. The device of stages is a method of representing a non-exponentially distributed state by a combination of stages each of which is exponentially distributed. The resulting model has constant transition rates and is easier to solve.

Application of the method of stages involves some difficulties:

1) Determination of a stage combination that will reasonably approximate the behaviour of a given distribution or fit the available data.

2) Determination of the parameters of this stage device.

The hazard rate is an important characteristic of a distribution. An awareness of its variability can be very useful in selecting a proper combination. The combinations discussed in this thesis can generate the following four basic hazard rate shapes:

1) Increasing hazard rate (Figure 3.1a)
2) Decreasing hazard rate (Figure 3.1b)
3) Initial period of decreasing hazard rate followed by increasing rate (Figure 3.1c)
4) Initial period of increasing hazard rate followed by decreasing rate (Figure 3.1d).

After selecting a proper combination, the parameters can be derived using the moment matching technique proposed in this thesis. This technique can be applied to both long term and short term reliability evaluations.

3.3 Stage Combination Models

The application of the method of stages commences with the selection of an appropriate stage combination model to approximate the given non-exponential distribution. The selection will depend on the characteristic behaviour of the distribution simulated, the required degree of accuracy, the simplicity of the model and the time range within which the approximation is required. The similarity between the transition rate function of the combination model and that of the distribution to be approximated is the major factor in the first choice of the approximate models considered in this thesis. The final choice of stage combinations is made by balancing the required accuracy and the model complexities. Careful selection of the initial model is important for another reason. The determination of the parameters of the approximate stage combination model tends to be difficult or even impossible if the model is not judiciously chosen.

The characteristics of distributions associated with combination models must be appreciated in order to select a suitable model. A number of stage combinations are introduced in this thesis and their characteristics are examined. Expressions and graphs for transition rate functions, probability density functions, and survivor functions for these models are presented.

1. A single stage

If the down time distribution is exponential, the down state can be represented by a single stage whose characteristics can be completely described by a constant transition rate ρ (Figure 3.2). It is characteristic only of the exponential distribution that the transition rate from a state does not depend on the residence time in it. A constant transition rate makes the process Markovian and the mathematical solution is much simpler than for other possible distributions. States having constant transition rates are the basic building blocks in the method of stage combination. An actual physical state may be represented by several stages each of which is associated with a constant transition rate. A stage in this thesis is, therefore, defined as a substate of a derived Markov process.

The probability distribution function of an exponential distribution is $\rho e^{-\rho x}$, and the survivor function is $e^{-\rho x}$. The mean and the standard deviation of this distribution are identical and equal to $1/\rho$.

2. Two identical stages in series

A down time duration which is represented by two identical

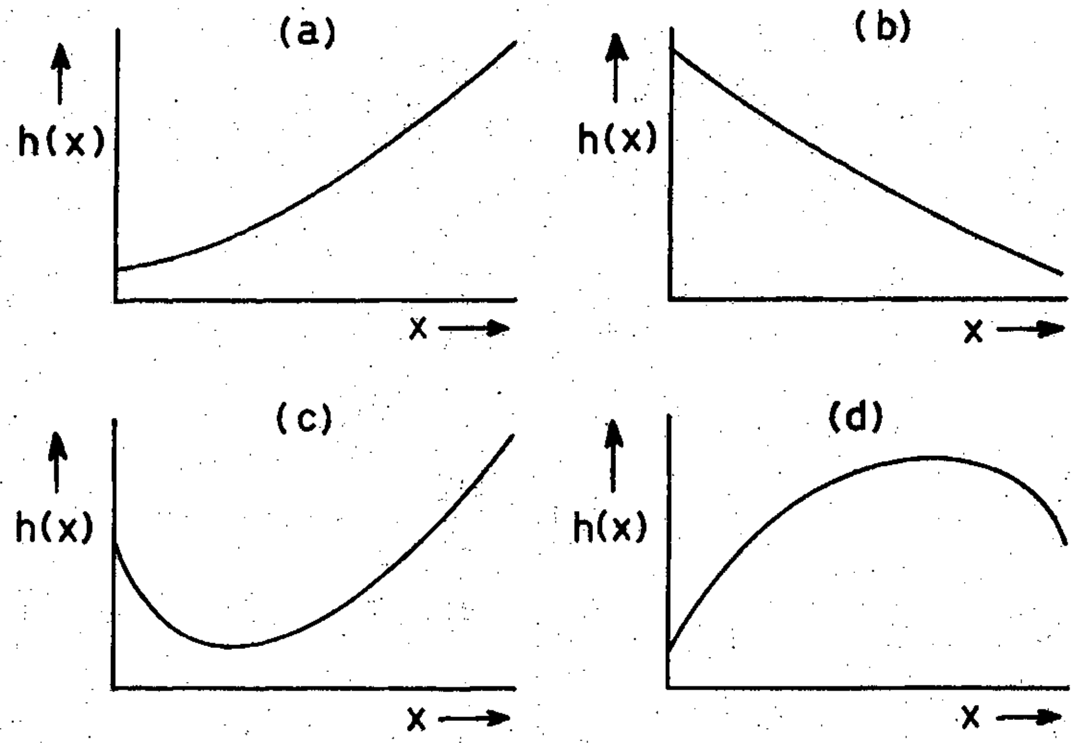


Figure 3.1: Some Typical Hazard Rate Functions.

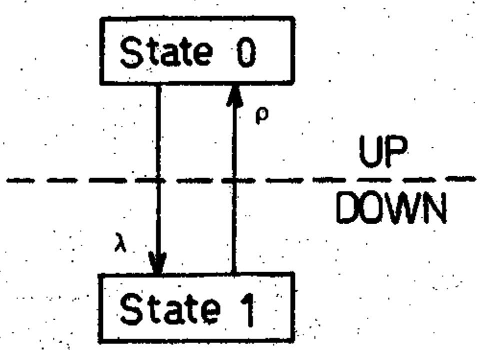


Figure 3.2: The State Transition Diagram for a System Whose Up Time and Down Time are Exponentially Distributed.

stages in series (Figure 3.3) is not exponentially distributed. The down time is the addition of two independent exponentially distributed random variables. The separation of the state into two substates with constant transition rates is solely for the convenience of representation. The substates may not have any actual physical significance.

The transition rate function becomes time dependent and is

$$\frac{\rho^2 x}{1 + \rho x}$$

where ρ is the transition rate for each substate. The transition rate approaches a constant ρ as the time tends to be long. The probability density function is $\rho^2 x e^{-\rho x}$ and the survivior function $e^{-\rho x}(1 + \rho x)$. The mean of this distribution is $2/\rho$. The standard deviation is $\sqrt{2}/\rho$.

3. A series of identical stages

If repair takes place in "a" identical stages, the down time distribution is then the special Erlangian distribution (Figure 3.4). The process takes "a" stages in the down state before it transfers to the up state. The times spent in these stages are independent and exponentially distributed. The number of stages "a" is one of the parameters for this form of combination which includes the single stage and the two identical stages in series noted previously as special cases of a=1 and a=2. As the transition rate of each state ρ and the number of states "a" are varied, the combination can approximate a range of distributions. The transition rate function for the special Erlangian distribution shows positive aging for all values of a>1 (Figure 3.5a). It approaches ρ after a long time. The probability density function is

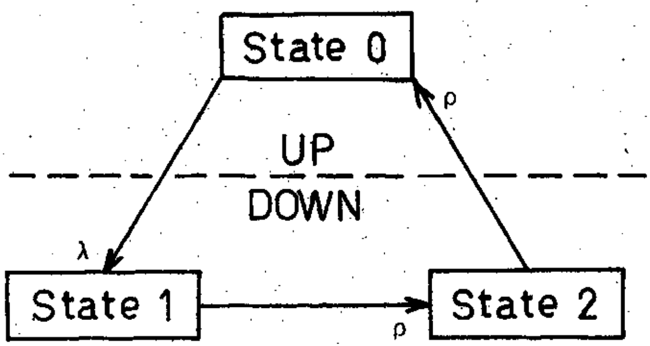


Figure 3.3: The State Transition Diagram of a System with the Down State Represented by Two Series Stages in Series.

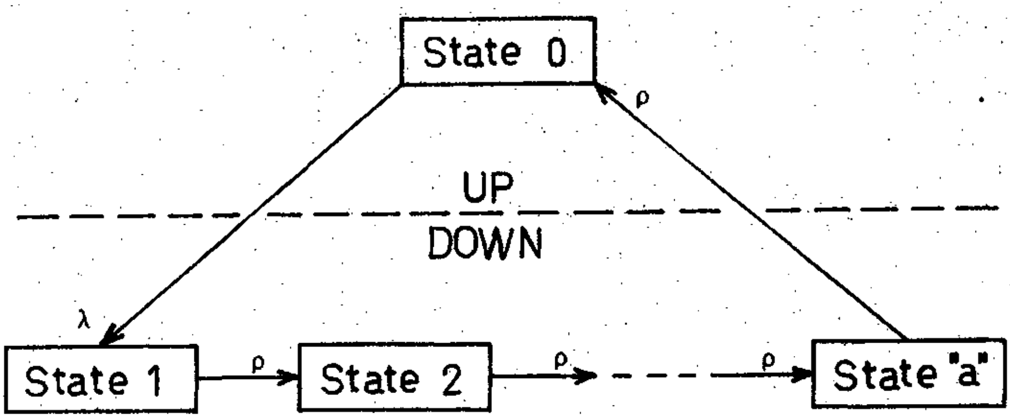


Figure 3.4: The State Transition Diagram of a System with the Down Time Represented by the Special Erlangian Distribution.

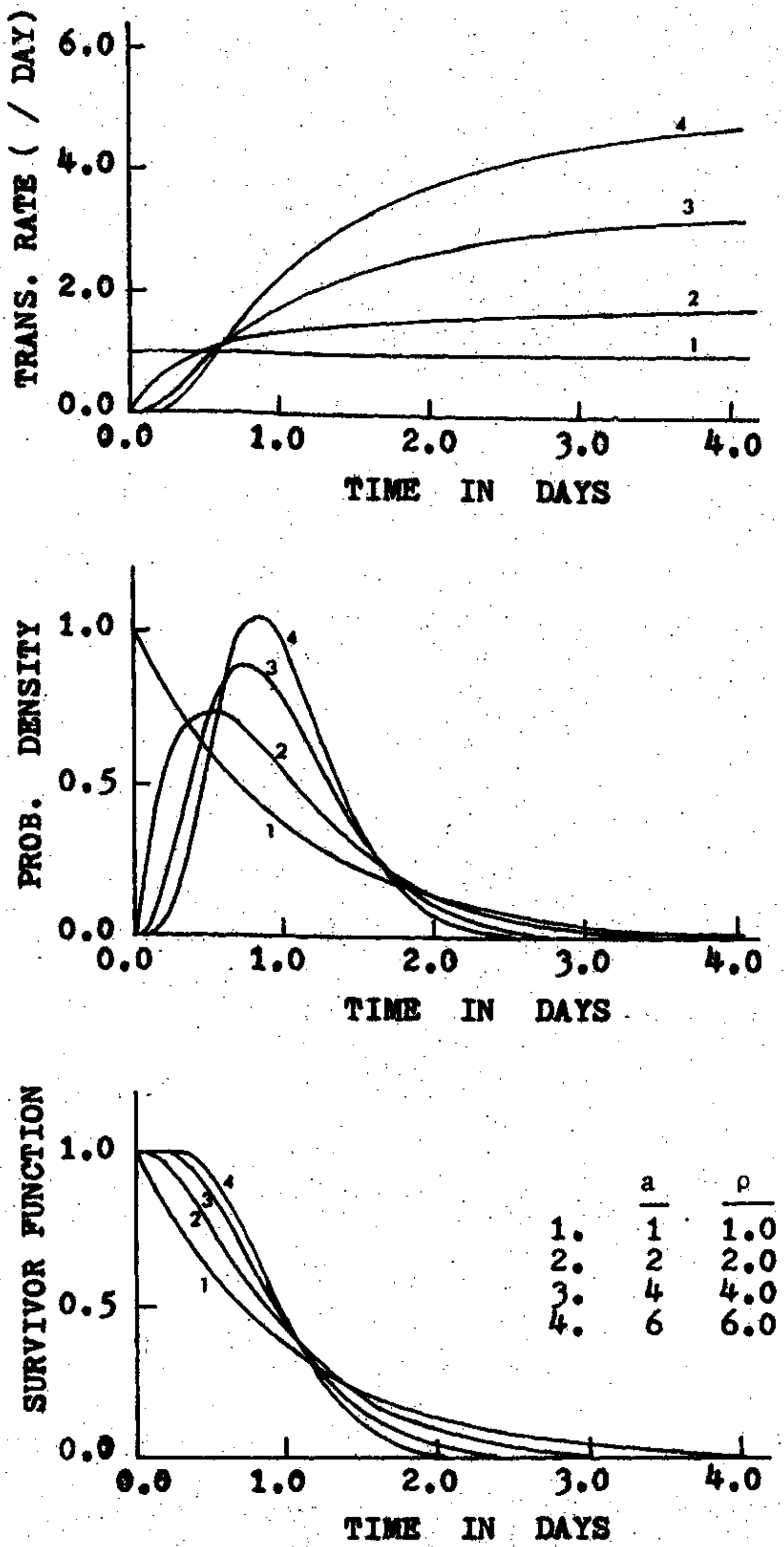


Figure 3.5: Characteristics of the Special Erlangian Distribution.

$$\frac{\rho(\rho x)^{a-1}\, e^{-\rho x}}{(a-1)!}$$

and the survivor function is

$$e^{-\rho x} \sum_{i=1}^{a} \frac{(\rho x)^{i-1}}{(i-1)!} .$$

The mean and standard deviation of this distribution are a/ρ and $\sqrt{a}/\rho$ respectively. The characteristics of a family of special Erlangian distributions with a constant mean time of one day are shown in Figure 3.5. This value was used throughout the chapter other than those shown in Figure 3.14. As "a" increases, the dispersion about the mean decreases, if the mean is kept constant. The approximation of the Weibull distribution by this combination is discussed in reference (11).

4. Two series stages in parallel

If two series stage combinations are connected in parallel with the probabilities of entering the first stages of ω_1, ω_2, this model can approximate a wider range of distributions than the special Erlangian distribution (Figure 3.6). The combination now has five parameters: ω_1, the transition rate of each upper stage (ρ_1); the transition rate of each lower stage (ρ_2); the number of stages for the upper series (a_1); the number of stages for the lower series (a_2). The parameter ω_2 can be found simply by subtracting ω_1 from 1.0.

This combination shows a wide range of transition rate functions (Figure 3.7a). The expression for the probability density func-

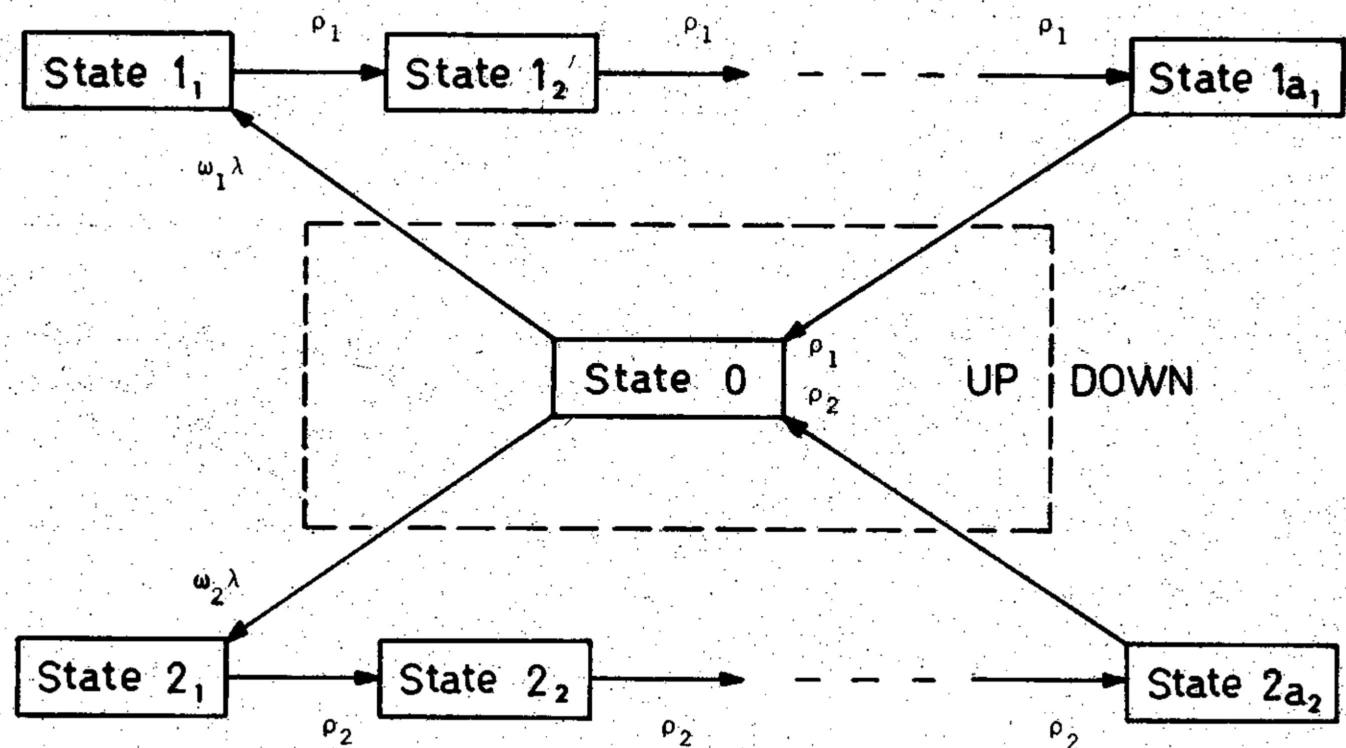
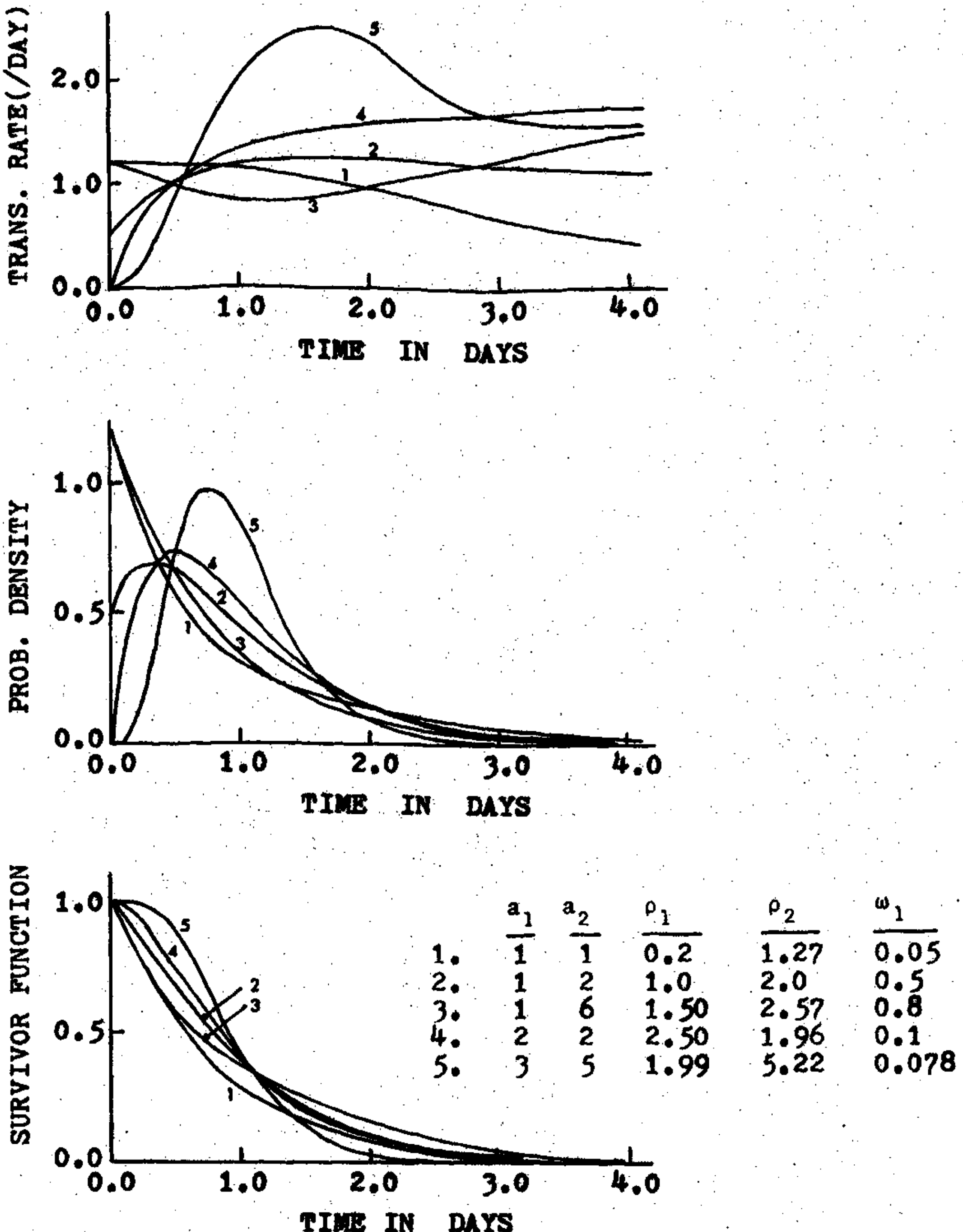


Figure 3.6: The State Transition Diagram of a System with the Down State Represented by Two Series Stage Combinations in Parallel.



	a_1	a_2	ρ_1	ρ_2	ω_1
1.	1	1	0.2	1.27	0.05
2.	1	2	1.0	2.0	0.5
3.	1	6	1.50	2.57	0.8
4.	2	2	2.50	1.96	0.1
5.	3	5	1.99	5.22	0.078

Figure 3.7: Some Characteristics of the Distribution Associated with the Series Combination of Two Special Erlangian Distributions.

tion is (Figure 3.7b):

$$\omega_1\rho_1 e^{-\rho_1 x} \frac{(\rho_1 x)^{a_1-1}}{(a_1-1)!} + \omega_2\rho_2 e^{-\rho_2 x} \frac{(\rho_2 x)^{a_2-1}}{(a_2-1)!} .$$

The survivor function is (Figure 3.7c):

$$\omega_1 e^{-\rho_1 x} \sum_{i=1}^{a_1} \frac{(\rho_1 x)^{i-1}}{(i-1)!} + \omega_2 e^{-\rho_2 x} \sum_{i=1}^{a_2} \frac{(\rho_2 x)^{i-1}}{(i-1)!} .$$

The mean of this distribution is:

$$\frac{\omega_1 a_1}{\rho_1} + \frac{\omega_2 a_2}{\rho_2} .$$

It has been noted previously that the fictitious stages representing an actual system state need not have any physical significance. It is, however, possible to give a physical interpretation to two series stages in parallel. It represents two types of repair occuring on the average $100\omega_1$, $100\omega_2$ percent of the total occurrences and each having a special Erlangian distribution.

5. Two different stages in series

The combination of two stages in series can be considered for the condition of two different transition rates ρ and ρ_1 (Figure 3.8). The transition rate function shows positive aging converging to the smaller of the two transition rates.

The transition rate function is:

$$\frac{\rho\rho_1 \ (e^{-\rho_1 x} - e^{-\rho x})}{\rho e^{-\rho_1 x} - \rho_1 e^{-\rho x}} \ .$$

The probability density function is:

$$\frac{\rho\rho_1}{\rho - \rho_1} \ (e^{-\rho_1 x} - e^{-\rho x}) \ .$$

The survivor function is:

$$\left(\frac{\rho}{\rho - \rho_1}\right) e^{-\rho_1 x} + \left(\frac{\rho_1}{\rho_1 - \rho}\right) e^{-\rho x} \ .$$

The mean of this distribution is $1/\rho + 1/\rho_1$.

6. Series stages in series with a distinctive stage

The special Erlangian distribution with its characteristic of stage by stage repair can be generalized by considering the distributions associated with each series stage to have different transition rates (reference 7), ρ_1, ρ_2, ---, ρ_a. This model generates a wider range of distributions than the special Erlangian distribution with little added mathematical complexity. In this thesis a special case of this situation, a series of identical stages in series with a distinctive stage, is examined (Figure 3.9). This model has three parameters: the transition rate ρ of each stage, the number of stages "a"

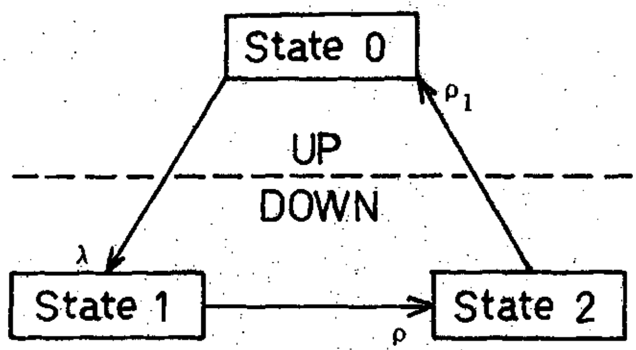


Figure 3.8: The State Transition Diagram of a System with the Down State Represented by Two Different Stages in Series.

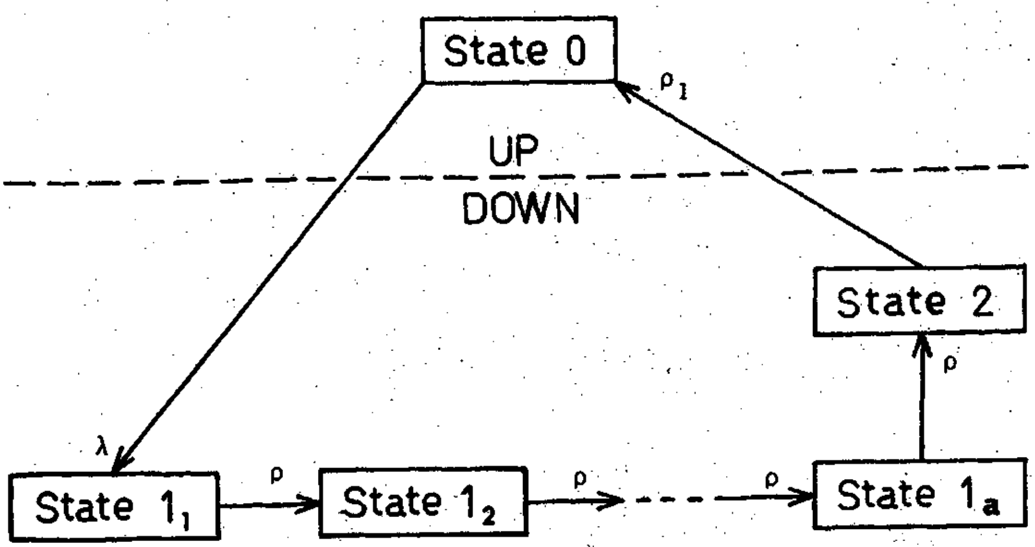


Figure 3.9: The State Transition Diagram of a System With the Down State Represented by "a" Number of Series Stages in Series with a Distinctive Stage.

in the identical series stages and the transition rate ρ_1 for the distinctive stage.

The transition rate function is shown in Figure 3.10a. The probability density function is:

$$\rho_1 \left(\frac{\rho}{\rho - \rho_1}\right)^a \left[e^{-\rho_1 x} - e^{-\rho x} \sum_{i=1}^{a} \frac{\{(\rho - \rho_1)x\}^{i-1}}{(i - 1)!}\right].$$

It is shown in Figure 3.10b. The survivor function is:

$$e^{-\rho x} \sum_{i=1}^{a} \frac{(\rho x)^{i-1}}{(i - 1)!} + \left(\frac{\rho}{\rho - \rho_1}\right)^a \left[e^{-\rho_1 x} - e^{-\rho x} \sum_{i=1}^{a} \frac{\{(\rho - \rho_1)x\}^{i-1}}{(i - 1)!}\right].$$

Some particular survivor functions are shown in Figure 3.10c. The mean of this distribution is $a/\rho + 1/\rho_1$.

7. <u>Series stages in series with two parallel stages</u>

This combination has a series of identical stages followed by two parallel stages with probabilities ω_1 and ω_2 as shown in Figure 3.11a. This is equivalent to two "series stages in series with a distinctive stage" in parallel (Figure 3.11b).

The combination has five parameters: "a" - the number of stages in the identical series stage, ω_1 - probability to the upper stage, ρ_1 - transition rate out of the upper stage, ρ_2 - transition rate out of the lower stage, ρ - transition rate from each state in the series stages. The probability density function can be expressed by:

$$f(x) = \omega_1 \rho_1 \left(\frac{\rho}{\rho - \rho_1}\right)^a \left[e^{-\rho_1 x} - e^{-\rho x} \sum_{i=1}^{a} \frac{\{(\rho - \rho_1)x\}^{i-1}}{(i - 1)!}\right]$$

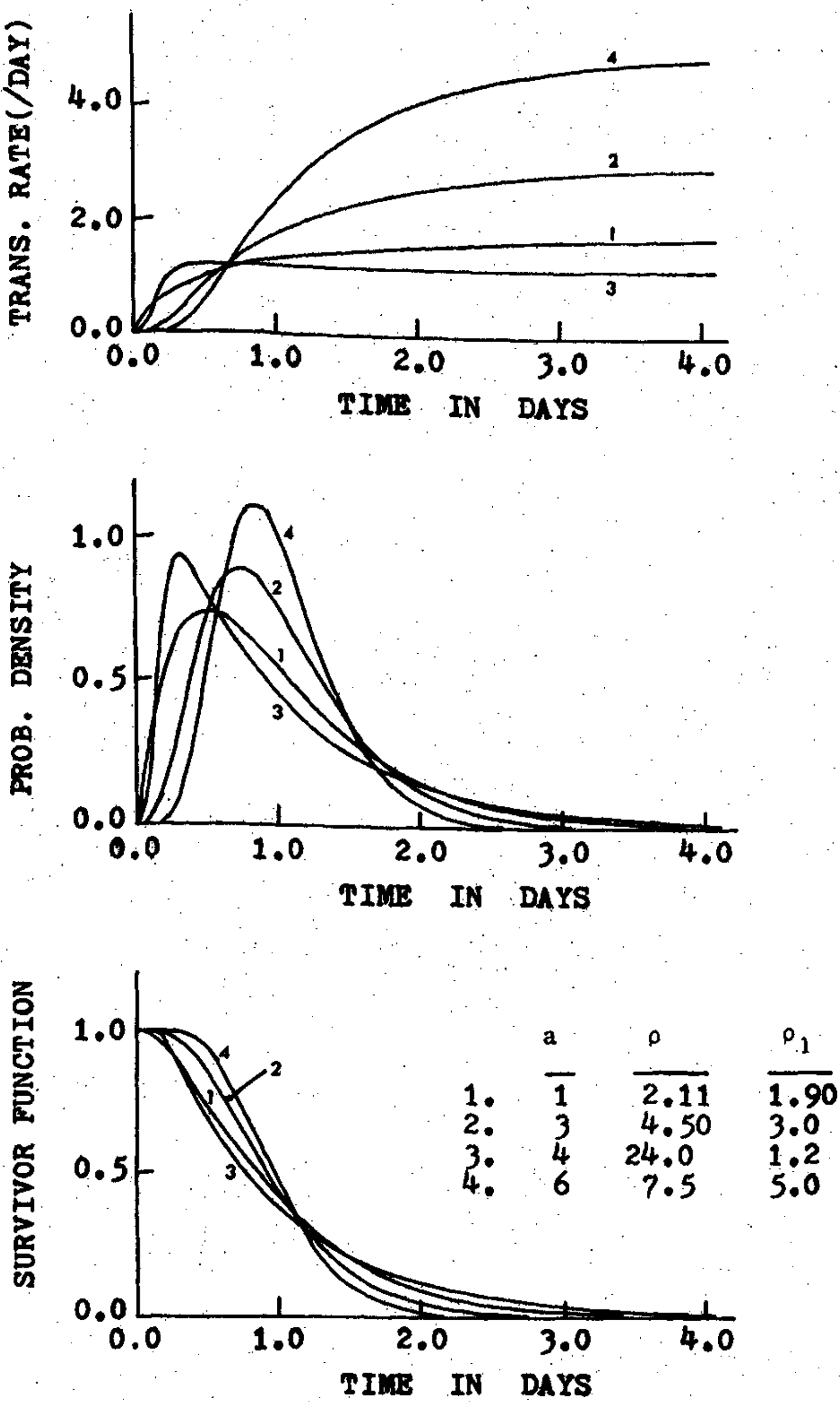
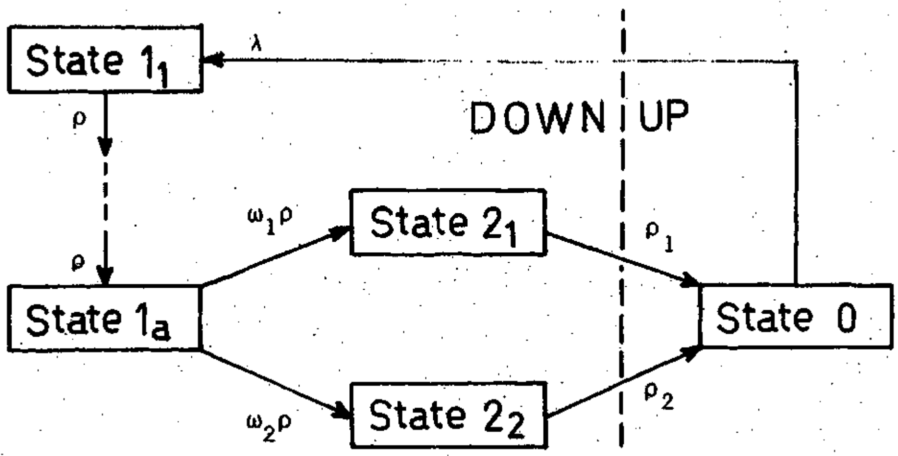
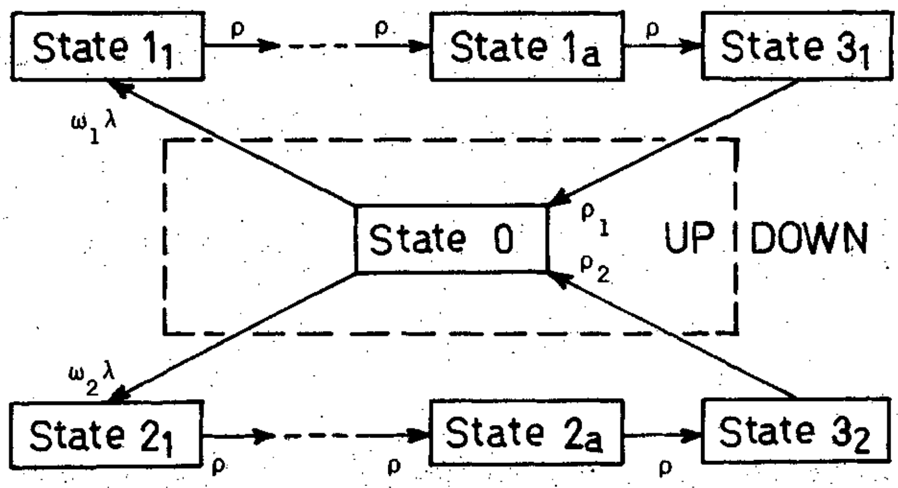


Figure 3.10: Some Characteristics of the Distribution Associated with Series Stages in Series with a Distinctive Stage.



(a) A Series Stages in Series with Two Parallel Stages.



(b) Two "Series Stages in Series with a Distinctive Stages" in Parallel.

Figure 3.11: The State Transition Diagram of a System with the Down State Represented by: (a) and (b).

$$+ \omega_2\rho_2 \left(\frac{\rho}{\rho - \rho_2}\right)^a \left[e^{-\rho_2 x} - e^{-\rho x} \sum_{i=1}^{a} \frac{\{(\rho - \rho_2)x\}^{i-1}}{(i-1)!}\right].$$

The survivor function can be expressed by:

$$F(x) = \sum_{i=1}^{a} \frac{(\rho x)^{i-1}}{(i-1)!} e^{-\rho x} + \omega_1 \left(\frac{\rho}{\rho - \rho_1}\right)^a \left[e^{-\rho_1 x} - e^{-\rho x} \sum_{i=1}^{a}\right.$$

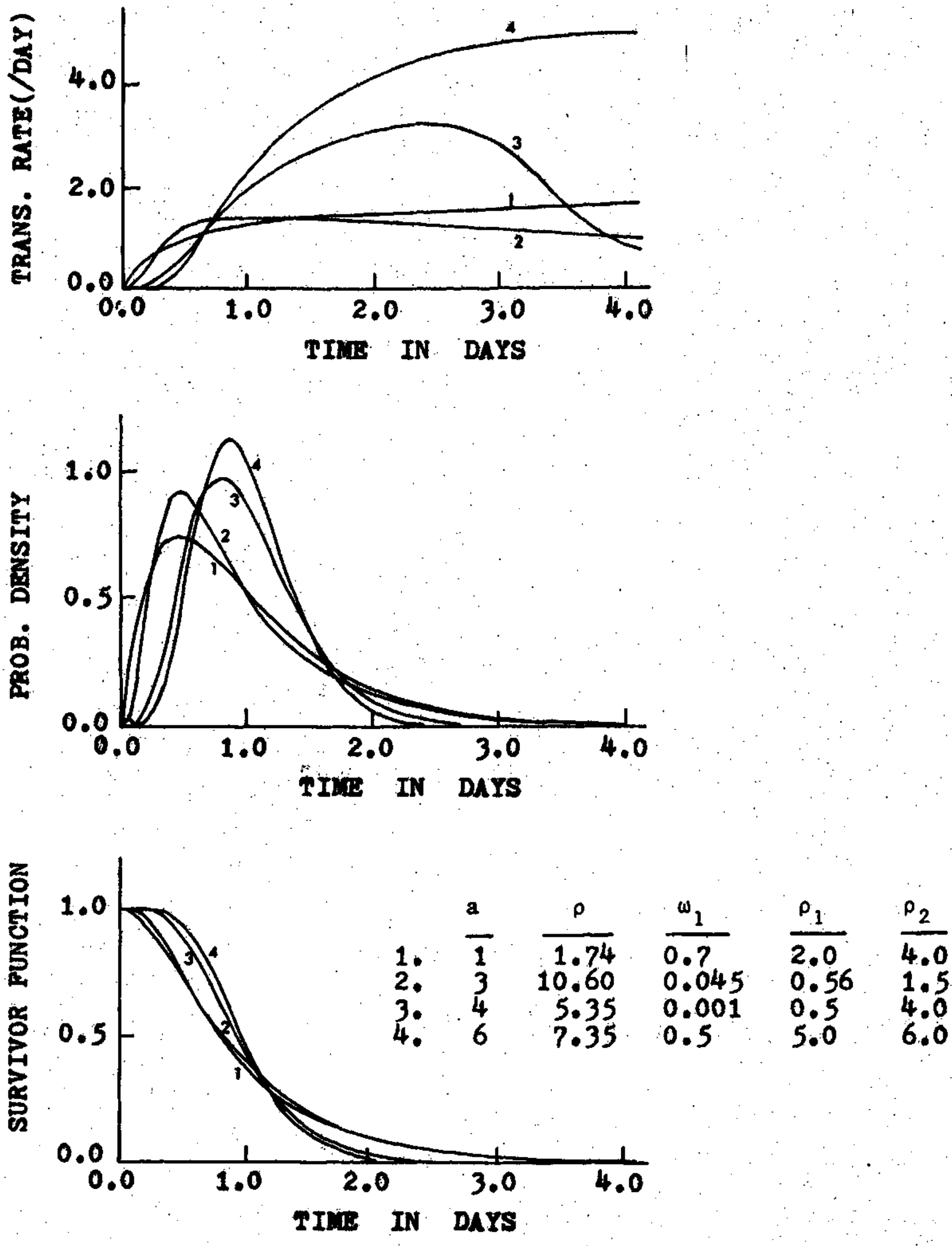
$$\left.\frac{\{(\rho - \rho_1)x\}^{i-1}}{(i-1)!}\right] + \omega_2 \left(\frac{\rho}{\rho - \rho_2}\right)^a \left[e^{-\rho_2 x} - e^{-\rho x} \sum_{i=1}^{a}\right.$$

$$\left.\frac{\{(\rho - \rho_2)x\}^{i-1}}{(i-1)!}\right].$$

The transition rate function can be found from $h(x) = f(x)/F(x)$. The mean of this distribution is $a/\rho + \omega_1/\rho_1 + \omega_2/\rho_2$. Some characteristics of the distribution are shown in Figure 3.12.

3.4 Determination of Parameters

After a model is chosen to approximate a distribution, the next problem is to find the parameters so that it will fit the distribution. There are generally no explicit formulae for directly deriving the approximate stage model parameters from those of the distribution. In many cases the parameters that will best define an empirical distribution are not known. The moments can, however, always be evaluated for any distribution either by exact or approximate methods. A method of determining parameters for approximate stage models by matching the first r moments of the model and the distribution is presented in the



	a	ρ	ω_1	ρ_1	ρ_2
1.	1	1.74	0.7	2.0	4.0
2.	3	10.60	0.045	0.56	1.5
3.	4	5.35	0.001	0.5	4.0
4.	6	7.35	0.5	5.0	6.0

Figure 3.12: Some Characteristics of the Distribution Associated with Series Stages in Series with Two Parallel Stages.

following sections. This method is quite general in application.

The parameters of the stage model are non-linear and implicit functions of its moments. On the contrary, the first r moments for the stage combinations discussed in this thesis can be easily calculated from the parameters. The Newton-Raphson method of successive approximation is applied to solve for the parameters from the given moments. This method requires the following steps for each stage of approximation:

1) evaluation of the moments with given parameters.
2) evaluation of the partial derivatives of the moments with respect to each parameter.

1. Moment evaluation for a stage combination

The probability density functions of the stage combinations discussed before have simple rational Laplace transforms. The r-th moment can be obtained from the r-th derivative of the Laplace of the p.d.f. In other words, the r-th moment, M^r, of a distribution can be obtained by $M^r = (-1)^r \bar{f}^{(r)}(0)$, where $\bar{f}(s)$ is the Laplace of the probability function, and

$$\bar{f}^{(r)}(0) = \left.\frac{d^r \bar{f}(s)}{ds^r}\right|_{s=0} .$$

Moment calculations for some stage models are given in Appendix 3.

2. Newton Raphson method for parameter calculation

The first r moments are matched by successive approximation starting from the initial parameters. If a model has r parameters, x_1, ---, x_r, to be determined by matching the first r moments, then the r functions $\phi_1, \phi_2, ---, \phi_r$ are defined such that:

$$\phi_1 = \phi_1(X) = M^1(X) - m^1$$

$$\phi_2 = \phi_2(X) = M^2(X) - m^2$$

$$---$$

$$\phi_r = \phi_r(X) = M^r(X) - m^r$$

where X is the column vector $(x_1 \ x_2 \ --- \ x_r)^t$ and $M^r(X)$ is the r-th moment of the stage model and m^r is the r-th moment of the distribution to be approximated. The condition of exact match of the first r moments are:

$$\phi_1 = \phi_2 = --- = \phi_r = 0.$$

Let $X_o = (x_{1o} \ x_{2o} \ --- \ x_{ro})^t$ be the parameter vector at a certain stage of approximation and let ϕ be a column vector such that $\phi = (\phi_1 \ \phi_2 \ --- \ \phi_r)^t$. The correction vector for parameters $\Delta X = (\Delta x_1 \ \Delta x_2 \ --- \ \Delta x_r)^t$ can be calculated from the following matrix equation by the Gauss elimination method if $\phi(X_o)$ and $\phi'(X_o)$ are known:

$$\phi(X_o) + \phi'(X_o) \cdot \Delta X = 0$$

where $\phi(X_o)$ is the vector ϕ at $X = X_o$ and $\phi'(X_o)$ is the Jacobian matrix of ϕ at X_o, i.e.,

$$\phi'(X_o) = \begin{bmatrix} \frac{\partial\phi_1}{\partial x_1}(X_o) & \frac{\partial\phi_1}{\partial x_2}(X_o) & --- & \frac{\partial\phi_1}{\partial x_r}(X_o) \\ \frac{\partial\phi_2}{\partial x_1}(X_o) & \frac{\partial\phi_2}{\partial x_2}(X_o) & --- & \frac{\partial\phi_2}{\partial x_r}(X_o) \\ & --- & & \\ \frac{\partial\phi_r}{\partial x_1}(X_o) & \frac{\partial\phi}{\partial x_2}(X_o) & --- & \frac{\partial\phi_r}{\partial x_r}(X_o) \end{bmatrix}$$

The improved parameter values will be obtained by $X = X_o + \Delta X$. $\phi(X_o)$ can be calculated directly from the first r moments of the model when $X = X_o$ (Appendix 3). The method of finding $\phi'(X_o)$ is discussed in Appendix 4 for some stage combinations.

3.5 Example System Study

The technique of the method of stages is applied to the two state generation system model shown in Figure 3.13. The exponential distribution is a reasonable assumption for the up-time distribution when the unit is operating within its useful life period. It has been suggested in several publications that the lognormal is possibly a better representation in certain cases for the down-time.

The lognormal distribution is, therefore, used for the down-time distribution in this example. The mean time to failure and the mean repair time of 1500 and 20 hours, respectively, are used. The state probabilities are calculated for each of the three cases of down-time standard deviation values: 10 hours, 14.14 hours, and 20 hours. The lognormal distribution is completely specified by its mean and

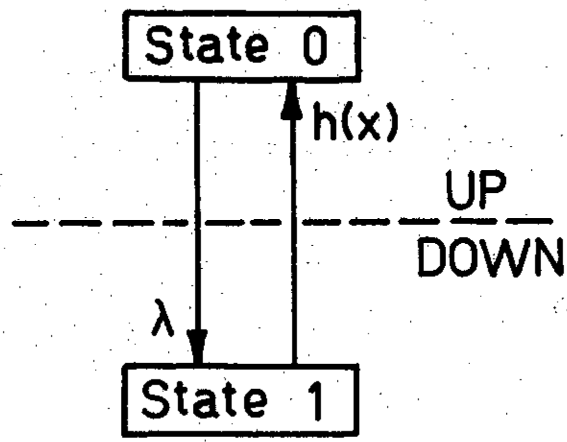


Figure 3.13: The State Space Diagram of the Two State Generation System Model with the Time Dependent Transition Rate from the Down State.

standard deviation.

Approximate expressions for calculating the probabilities for the two state model with the Weibull as the down-time distribution were developed in reference 8. This method is extended in this thesis for general down-time distributions including those which do not have closed form survivor functions. The results by this method are compared with those obtained using approximate stage combination models.

1. Lognormal distribution

If the logarithm of a random variable has a normal distribution, this random variable has a continuous distribution known as the lognormal distribution. This distribution describes a random process that represents the product of several independent events. The lognormal distribution has a family of probability functions described by:

$$\frac{1}{x\sqrt{2\pi}\ \alpha} \exp \left[- \frac{\{\log(x-\rho)\}^2}{2\alpha^2}\right]$$

where ρ and α are the mean and the standard deviation of the logarithm of the random variable of this distribution. The two parameters, ρ and α, of a lognormal distribution have the following relationships with its mean μ, and its standard deviation σ:

$$\rho = \log \mu - \tfrac{1}{2} \log \left[\left(\frac{\sigma}{\mu}\right)^2 + 1\right]$$

$$\alpha^2 = \log \left[\left(\frac{\sigma}{\mu}\right)^2 + 1\right].$$

The closed form expressions for the transition rate function and the survivor function do not exist. The transition rate function of the lognormal distribution shows initial positive aging followed by negative aging. The transition rate converges to zero after a long time.

2. Approximation of lognormal distribution by the method of stages

Examination of the transition rate function shows that it is possible to use "two series stages in parallel" combination or "series stages in series with two parallel stages" combination. The r-th moment is given by:

$$M^r = \exp\left(\rho r + \tfrac{1}{2}\alpha^2 r^2\right).$$

Lognormal distributions with standard deviations of 10, 14.14, and 20 hours are approximated by stage combinations. The parameters are listed in Table 3.1. The transition rate functions, probability density functions, and the survivor functions of the lognormal distributions together with those of the approximate models are shown in Figure 3.14.

The down state probability can be found from the approximate Markov models. Several techniques exist for evaluating time specific probabilities of the states of a Markovian process. The down state probability was evaluated up to 24 hours assuming the system was initially in the up state. The continuous time Markov process is approximated by a discrete time one using a small time interval and the state probabilities are obtained by multiplication of the transition probability matrix. The results are shown in Table 3.2.

Table 3.1: The Parameters of Stage Combinations for the Approximation of Lognormal Distributions

The mean of the distribution μ = 20 hours.
σ = standard deviation of the distribution.

(a) Approximation by series stages in series with two parallel stages

Lognormal	Approximate model parameters				
σ (hours)	a	ω_1	ρ (per hour)	ρ_1 (per hour)	ρ_2 (per hour)
10	6	0.10976	0.526715	0.078325	0.123519
14.14	4	0.05270	0.554433	0.036597	0.083496
20	3	0.02250	1.221384	0.016180	0.060515

(b) Approximation by two series stages in parallel

Lognormal	Approximate model parameters				
σ (hours)	a_1	a_2	ω_1	ρ_1 (per hour)	ρ_2 (per hour)
10	4	6	0.21601	0.144001	0.336001
14.14	2	4	0.25464	0.066666	0.241201
20	2	2	0.06699	0.031698	0.118301

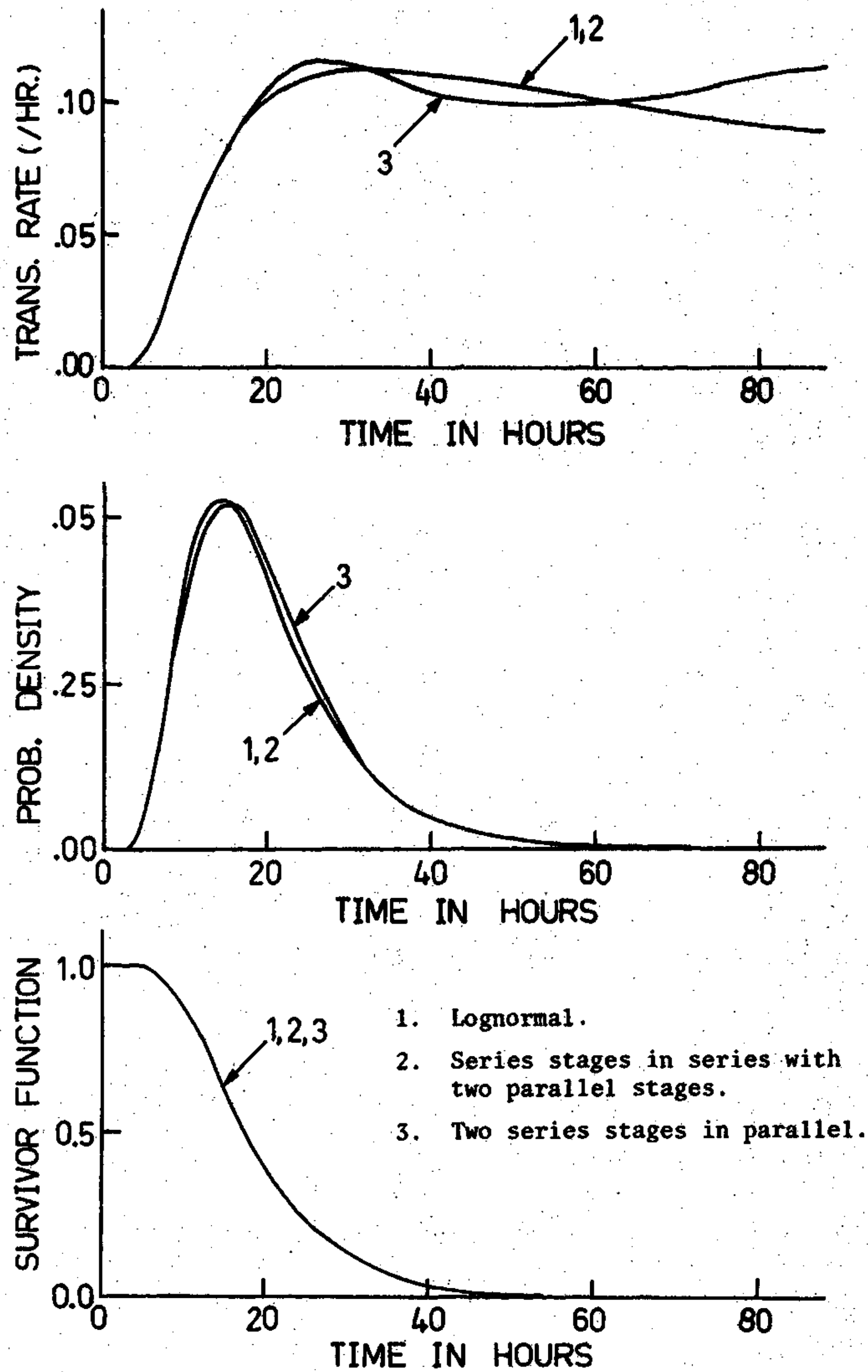


Figure 3.14: Characteristics of Lognormal Distributions and their Approximate Stage Combination Models (a) when μ = 20 hours and σ = 10 hours,

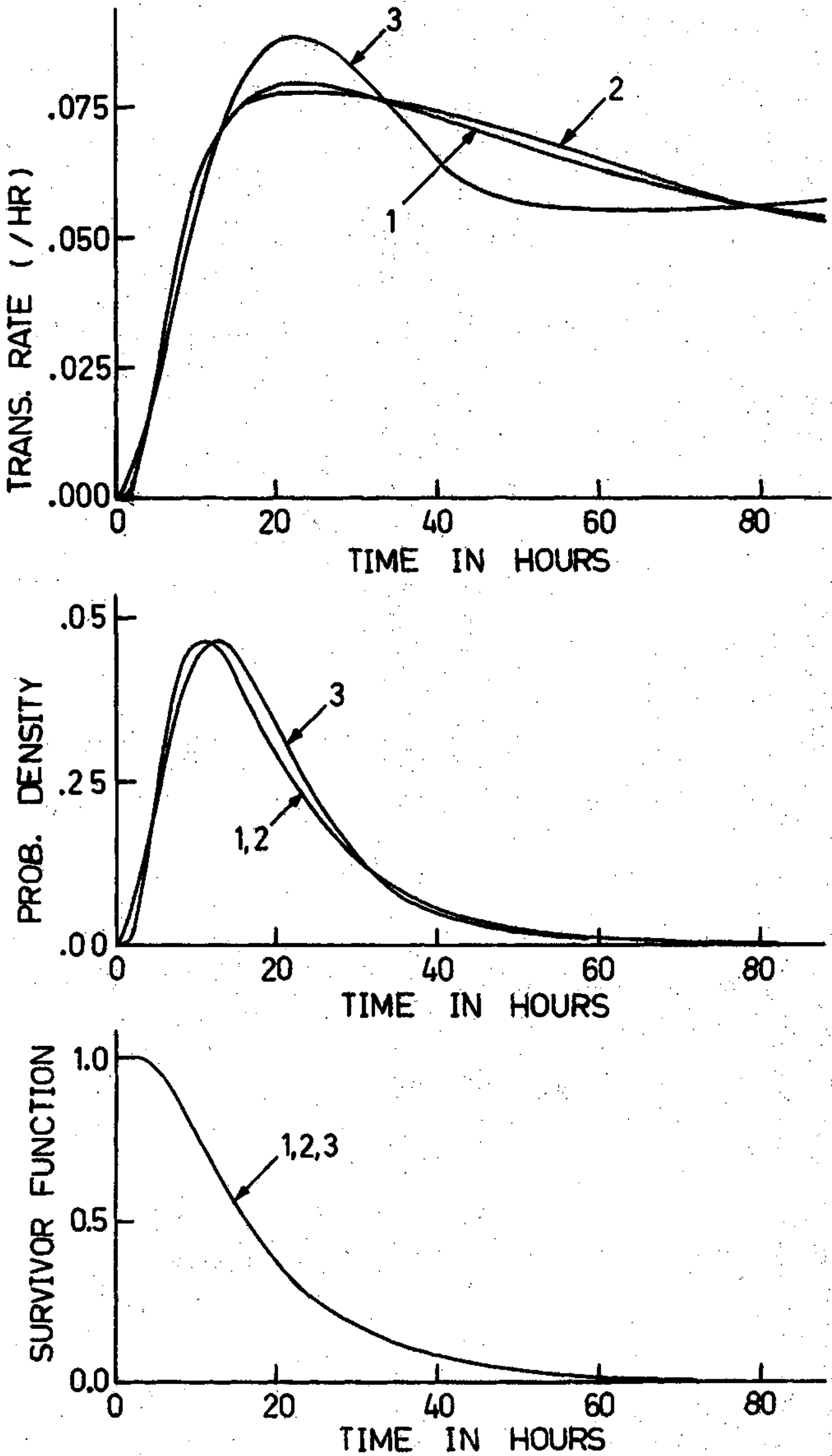


Figure 3.14: (continued)
(b) when μ = 20 hours and σ = 14.14 hours.

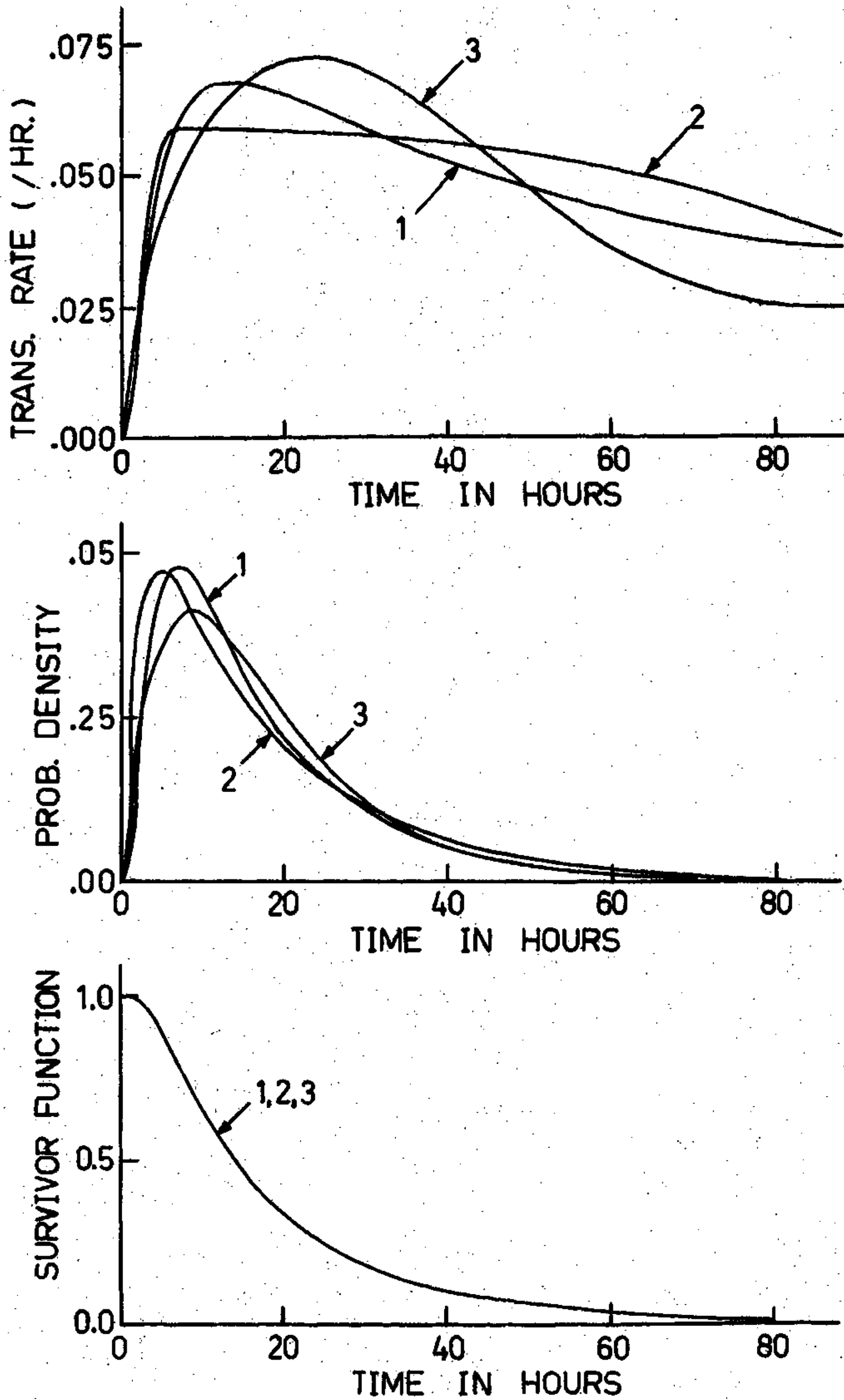


Figure 3.14: (continued)
(c) when μ = 20 hours and σ = 20 hours.

Table 3.2

Time specific probabilities of the down state calculated by the methods: a) the approximate expression; b) the approximate "series stages in series with two parallel stages"; c) the approximate "two series stages in parallel". The mean down time = 20 hours. The mean up time = 1,500 hours.

Case 1. Standard deviation = 10 hours

Time (hours)	Method Used (a)	(b)	(c)
1	6.667×10^{-4}	6.667×10^{-4}	6.667×10^{-4}
2	1.333×10^{-3}	1.334×10^{-3}	1.334×10^{-3}
3	2.000×10^{-3}	2.000×10^{-3}	2.000×10^{-3}
4	2.666×10^{-3}	2.666×10^{-3}	2.666×10^{-3}
5	3.332×10^{-3}	3.333×10^{-3}	3.328×10^{-3}
6	3.994×10^{-3}	3.988×10^{-3}	3.986×10^{-3}
7	4.650×10^{-3}	4.640×10^{-3}	4.635×10^{-3}
8	5.295×10^{-3}	5.280×10^{-3}	5.273×10^{-3}
9	5.923×10^{-3}	5.903×10^{-3}	5.895×10^{-3}
10	6.530×10^{-3}	6.505×10^{-3}	6.496×10^{-3}
11	7.110×10^{-3}	7.081×10^{-3}	7.072×10^{-3}
12	7.660×10^{-3}	7.628×10^{-3}	7.620×10^{-3}
13	8.178×10^{-3}	8.143×10^{-3}	8.137×10^{-3}
14	8.661×10^{-3}	8.623×10^{-3}	8.621×10^{-3}
15	9.109×10^{-3}	9.068×10^{-3}	9.070×10^{-3}
16	9.522×10^{-3}	9.477×10^{-3}	9.483×10^{-3}
17	9.901×10^{-3}	9.852×10^{-3}	9.863×10^{-3}
18	1.025×10^{-2}	1.019×10^{-2}	1.021×10^{-2}
19	1.056×10^{-2}	1.050×10^{-2}	1.052×10^{-2}
20	1.085×10^{-2}	1.078×10^{-2}	1.080×10^{-2}
21	1.111×10^{-2}	1.104×10^{-2}	1.106×10^{-2}
22	1.134×10^{-2}	1.126×10^{-2}	1.129×10^{-2}
23	1.155×10^{-2}	1.146×10^{-2}	1.149×10^{-2}
24	1.174×10^{-2}	1.165×10^{-2}	1.167×10^{-2}

Table 3.2 (Continued)

Case 2. Standard deviation = 14.14 hours

Time (hours)	Method Used (a)	(b)	(c)
1	6.667×10^{-4}	6.674×10^{-4}	6.670×10^{-4}
2	1.333×10^{-3}	1.334×10^{-3}	1.332×10^{-3}
3	2.000×10^{-3}	1.998×10^{-3}	1.995×10^{-3}
4	2.660×10^{-3}	2.658×10^{-3}	2.651×10^{-3}
5	3.312×10^{-3}	3.308×10^{-3}	3.297×10^{-3}
6	3.950×10^{-3}	3.943×10^{-3}	3.930×10^{-3}
7	4.567×10^{-3}	4.557×10^{-3}	4.545×10^{-3}
8	5.160×10^{-3}	5.147×10^{-3}	5.139×10^{-3}
9	5.725×10^{-3}	5.709×10^{-3}	5.708×10^{-3}
10	6.260×10^{-3}	6.240×10^{-3}	6.249×10^{-3}
11	6.765×10^{-3}	6.740×10^{-3}	6.761×10^{-3}
12	7.238×10^{-3}	7.208×10^{-3}	7.243×10^{-3}
13	7.680×10^{-3}	7.644×10^{-3}	7.694×10^{-3}
14	8.092×10^{-3}	8.050×10^{-3}	8.114×10^{-3}
15	8.475×10^{-3}	8.427×10^{-3}	8.503×10^{-3}
16	8.830×10^{-3}	8.776×10^{-3}	8.862×10^{-3}
17	9.160×10^{-3}	9.099×10^{-3}	9.193×10^{-3}
18	9.464×10^{-3}	9.398×10^{-3}	9.498×10^{-3}
19	9.746×10^{-3}	9.674×10^{-3}	9.776×10^{-3}
20	1.001×10^{-2}	9.993×10^{-3}	1.003×10^{-2}
21	1.025×10^{-2}	1.017×10^{-2}	1.026×10^{-2}
22	1.047×10^{-2}	1.038×10^{-2}	1.048×10^{-2}
23	1.067×10^{-2}	1.058×10^{-2}	1.067×10^{-2}
24	1.085×10^{-2}	1.077×10^{-2}	1.085×10^{-2}

Table 3.2 (Continued)

Case 3. Standard deviation = 20 hours

Time (hours)	Method Used (a)	(b)	(c)
1	6.666×10^{-4}	6.680×10^{-4}	6.654×10^{-4}
2	1.331×10^{-3}	1.330×10^{-3}	1.323×10^{-3}
3	1.985×10^{-3}	1.974×10^{-3}	1.966×10^{-3}
4	2.620×10^{-3}	2.591×10^{-3}	2.590×10^{-3}
5	3.230×10^{-3}	3.178×10^{-3}	3.192×10^{-3}
6	3.811×10^{-3}	3.732×10^{-3}	3.770×10^{-3}
7	4.362×10^{-3}	4.255×10^{-3}	4.321×10^{-3}
8	4.880×10^{-3}	4.478×10^{-3}	4.846×10^{-3}
9	5.367×10^{-3}	5.212×10^{-3}	5.342×10^{-3}
10	5.823×10^{-3}	5.650×10^{-3}	5.812×10^{-3}
11	6.250×10^{-3}	6.061×10^{-3}	6.254×10^{-3}
12	6.649×10^{-3}	6.449×10^{-3}	6.669×10^{-3}
13	7.022×10^{-3}	6.815×10^{-3}	7.059×10^{-3}
14	7.370×10^{-3}	7.159×10^{-3}	7.424×10^{-3}
15	7.696×10^{-3}	7.483×10^{-3}	7.765×10^{-3}
16	8.000×10^{-3}	7.789×10^{-3}	8.083×10^{-3}
17	8.285×10^{-3}	8.077×10^{-3}	8.380×10^{-3}
18	8.551×10^{-3}	8.348×10^{-3}	8.657×10^{-3}
19	8.800×10^{-3}	8.604×10^{-3}	8.915×10^{-3}
20	9.034×10^{-3}	8.845×10^{-3}	9.155×10^{-3}
21	9.253×10^{-3}	9.072×10^{-3}	9.378×10^{-3}
22	9.458×10^{-3}	9.286×10^{-3}	9.586×10^{-3}
23	9.650×10^{-3}	9.488×10^{-3}	9.778×10^{-3}
24	9.831×10^{-3}	9.678×10^{-3}	9.958×10^{-3}

3. The approximate expression for down state probability

The time specific down state probability was obtained by another method without using the method of stages so that the results obtained by using the method of stages could be compared with the results obtained by a completely different approach. The method retains the original generation system model and develops an approximate expression for the down state probability.

The down state probability in the two state model can be approximately (reference 8) given by:

$$P_2(x) \simeq \lambda\delta \sum_{i=1}^{k} F\{(k - i)\delta\}.$$

It is equivalent to the equation:

$$P_2(x) \simeq \lambda\delta \sum_{i=1}^{k} F\{(i - 1)\delta\},$$

where F is the survivor function of the down time distribution, k is the number of sub-intervals, δ is the length of each interval. A large k can be chosen for better accuracy. If so, δ becomes small because the time x must be equal to $k\delta$.

Since an explicit expression for the survivor function of the lognormal distribution does not exist, Simpson's rule was applied to calculate the survivor function by integrating the probability density function. If the time range in consideration is divided into N number of intervals at the end of which the down state probabilities are to be calculated, then the following approximate method can be developed to calculate the probabilities successively:

$$F(i\delta) = F\{(i-1)\delta\} - \frac{\delta}{6\ell} \sum_{j=o}^{\ell-1} [f\{(i-1)\delta + \frac{j\delta}{\ell}\} + 4f\{(i-1)\delta$$

$$+ (j + \tfrac{1}{2}) \frac{\delta}{\ell}\} + f\{(i-1)\delta + (j+1) \frac{\delta}{\ell}\}].$$

The method was developed for computer application without requiring excessive storage. Down state probabilities are evaluated up to 24 hours with one hour intervals and are listed in Table 3.2, together with those obtained by the method of stages. It can be observed that the results of the two approximate methods are quite close to each other. The method of stages can be used to calculate state probabilities at the time much greater than 24 hours. The other method, however, can only be applied for a short initial time period.

3.6 Conclusion

The use of the method of stages in solving non-Markovian reliability models has been discussed in this chapter. Several simple stage combinations have been proposed and their characteristics derived and discussed. One of these configurations can be used as the starting point or the first approximation to a given distribution or set of data. This approximation will be sufficient in most cases. Further refinements could, however, be made by suitable alterations. The derivation of the stage model parameters is the most important step in the approximation. Techniques based on moment matching are proposed for determining those values. Once this approximation has been made, further solution becomes relatively simple because the approximate model is Markovian.

The approximate expression for the two state model with Weibull as the repair time distribution proposed in reference (8) is extended to the general case when the survivor function may not have a closed form expression. The derivation of approximate expressions becomes increasingly difficult as the number of the states of the model increases and the interstate transition rates become complicated. The state probabilities obtained by this technique are compared with those by the method of stages. Both results are very similar.

The method of stages can be applied to a general reliability problem. The approximation is first made for each non-exponential distribution and then the entire approximate model can be developed. The final model becomes Markovian. The analysis of a Markovian process is relatively easy and well systematized. The approximate model can be applied to both long-term and short-term reliability studies. Though the explicit mathematical expressions may not emerge, the numerical solution can usually be obtained. The advantages of the method of stages are apparent for modelling multistate systems with complicated interstate transition rates. The method of stages is not a universal solution for non-Markovian models, but it is a definitely powerful tool.

4. SIMULATION METHOD

4.1 Introduction

Several basic techniques for reliability evaluation in complex systems have been discussed in the previous chapter. The approaches are quite precise because the overall behaviour of the system process is mathematically described and the desired indices are obtained by analytical methods. The simulation method or Monte Carlo method discussed in this chapter is basically different from these analytical approaches. In the simulation technique, state histories of a process are simulated and the reliability indices are obtained by averaging the outputs of these state histories. A state history is the description of how the system states change with time. There are infinite numbers of possible state histories in a process. The results obtained by simulation methods are not as precise as those obtained using the techniques previously described because only a finite number of the possible state histories can be generated by the Monte Carlo method. The analytical method is preferable to the simulation method. In some cases, however, the simulation method is the only technique which will provide a solution to the reliability problem.

The simulation technique has been applied to several different examples in this chapter to obtain long-term or short-term reliability indices. There is not a general simulation model that can be applied to all sorts of problems and many simulation models are possible for a given problem. The simulation method used is, therefore, explained for each of the problems presented in this chapter instead of trying to generalize the simulation technique.

An electronic digital computer is normally used for fast simulation. One of the most important aspects of the simulation technique is efficiency and, therefore, this is discussed in some detail in this chapter. The efficiency can be used to compare two different simulation methods when applied to the same problem.

4.2 Efficiency and Standard Error

If n number of samplings, x_1, x_2, ---, x_n have been tried, the mean, $\bar{x}$, of the samples and the standard deviation of the mean, s, are given by (reference 15):

$$\bar{x} = \sum_{i=1}^{n} x_i/n \qquad \text{and} \qquad s = \frac{1}{n}\sqrt{\frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n-1}} .$$

The standard deviation of a result obtained by simulation is designated as "standard error" in this chapter. The simulation has to be repeated several times to obtain the standard error. The calculation of standard errors requires little added computation time compared to the actual simulation times.

The standard error is a measure of the precision of the result. It can be decreased either by increasing the number of samplings or by adopting a better simulation method. The number of samplings is approximately proportional to the computer time for simulation. To decrease the standard error by k times, the simulation time must be increased by

k^2 times for a particular simulation method. The efficiency of a simulation method is roughly proportional to the inverse of the product of the square of the standard error and the simulation time. The efficiency of a simulation method does not improve with increased simulation time.

Two simulation methods can be compared as to their efficiencies. The efficiency gain of Method 2 relative to Method 1 is (reference 16):

$$\frac{\text{simulation time of 1}}{\text{simulation time of 2}} \cdot \left(\frac{\text{standard error of 1}}{\text{standard error of 2}}\right)^2 .$$

This relative efficiency is used later in this chapter to compare two different simulation methods for a given problem.

4.3 Application to Long-term Reliability Evaluation

The simulation method has been applied to the following two example systems. The first one is a Markovian system and the second is non-Markovian.

Example 1

The system is a three phase bank composed of single phase transformers with one spare unit (Figure 4.1). The repair is not restricted. It has been assumed that the failure, repair and installation rates are constant with respect to time. The system becomes Markovian under this assumption.

The following system parameter values were used:

λ = failure rate = 0.25 failures/year for each unit

μ = repair rate = 12.0 repairs/year

γ = installation rate = 182.5 installations/year.

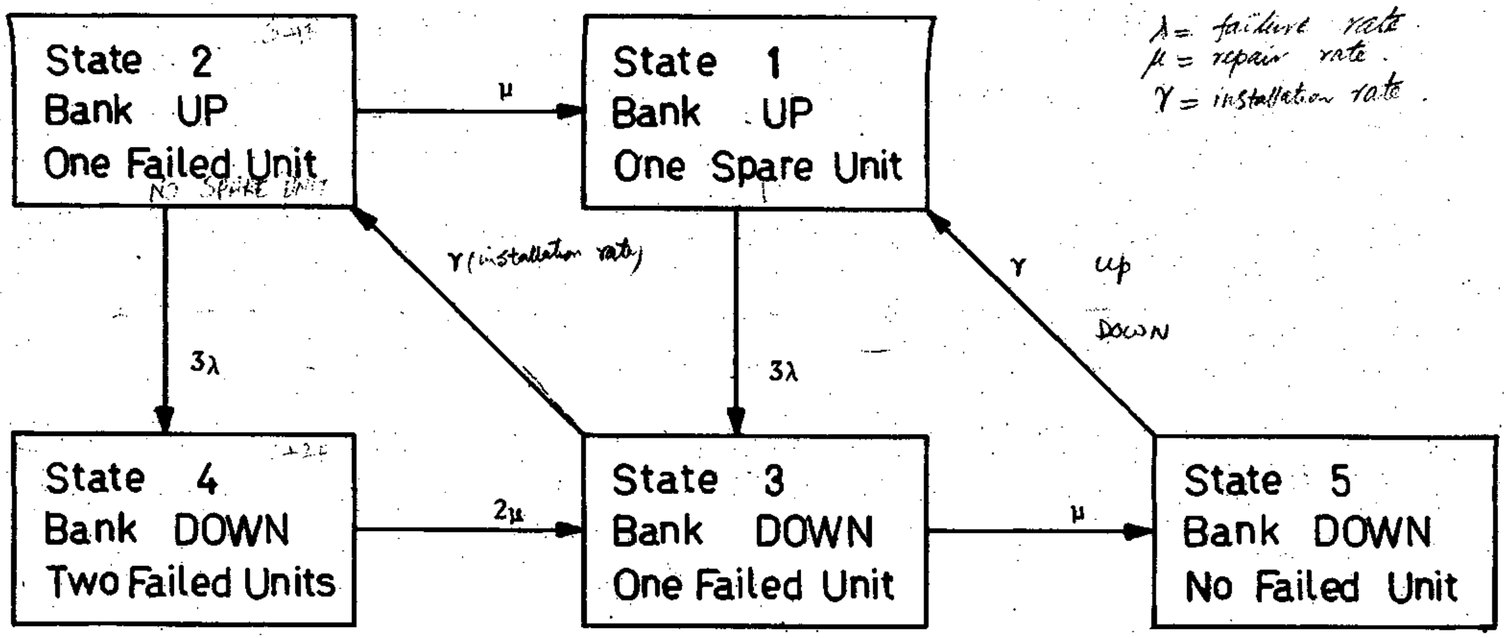


Figure 4.1: State Space Diagram for a 3-phase Transformer Bank Composed of Single Phase Transformers with One Spare Unit.

The simulation of this system is a generation of the state histories. A possible state history is shown in Figure 4.2. If a simulated state history is long enough, steady state reliability values can be estimated from it.

The following steps are repeated to generate a state history: the generation of time duration in the present state, and the determination of the next state. After the next state has been chosen, the current simulation time is increased by the amount of the time duration in the present state. Then, the next state becomes the present state for the next step.

A) Generation of the Time Duration in the Present State

The time duration in a state is exponentially distributed. If the system is in the present state and it has n numbers of possible next states with λ_i as the transition rate from the present state to the next i-th state as shown in Figure 4.3, then the time duration x in the present state is distributed such that it has the probability density function:

$$f(x) = \lambda e^{-\lambda x} \quad \text{where} \quad \lambda = \sum_{i=1}^{n} \lambda_i .$$

The cumulative probability function of x can be expressed by:

$$F(x) = 1 - e^{-\lambda x} .$$

A random variable, η, with known inverse of its cumulative probability function, F^{-1}, can be generated from a uniformly distributed

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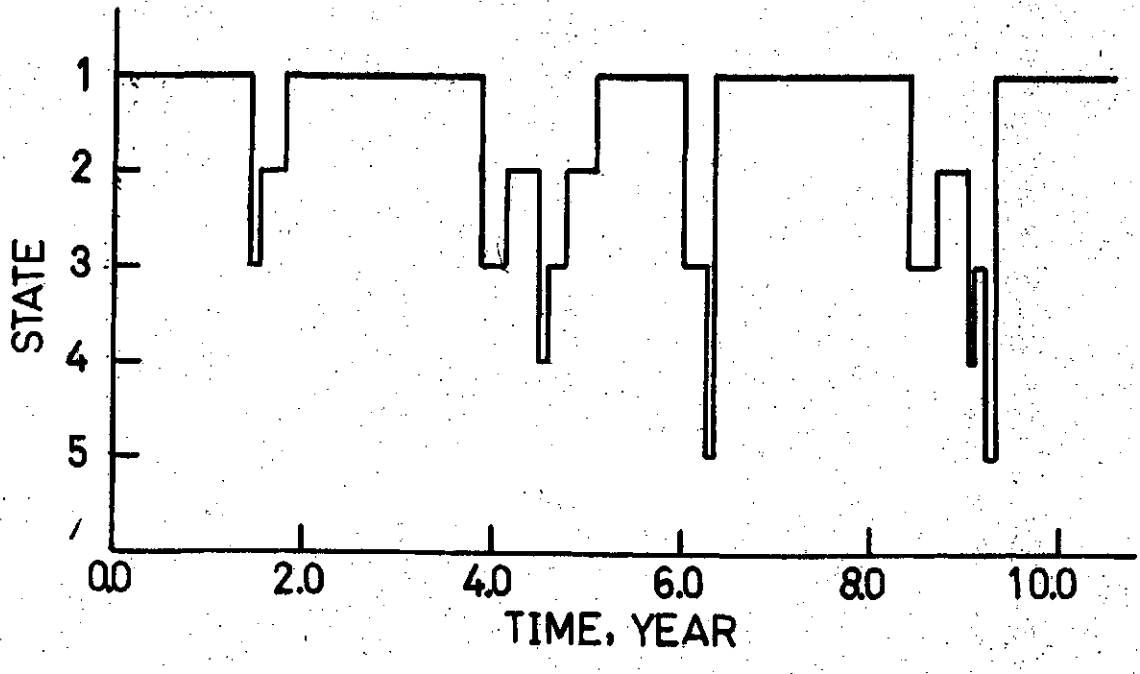

Figure 4.2: A Possible State History of the Three Phase Transformer Bank System.

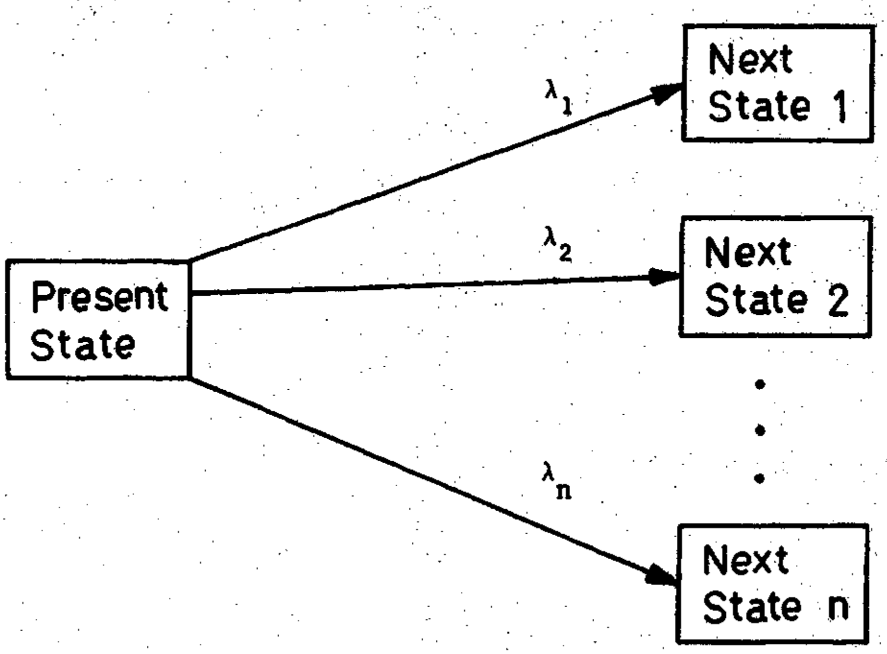

Figure 4.3: Present State with n Number of Next Possible States.

random number ξ ranging from 0 to 1 by taking $\eta = F^{-1}(\xi)$. FORTRAN SUBROUTINE RANDU produces a uniformly distributed pseudo-random number between 0 and 1 with a period of 2^{29} terms (reference 13). This subroutine was used to generate all the random variables discussed in this chapter. The time duration x with exponential distribution can be generated from the uniformly distributed random variable ξ by:

$$x = (-\log \xi) / \lambda .$$

B) Determination of the Next State

The probability of transition from the present state to the next k-th state given that the present state has been over is λ_k/λ. If a uniformly distributed random number ξ is generated and if the k-th state is chosen as the next state when

$$\sum_{i=1}^{k-1} \lambda_i < \lambda\xi \leq \sum_{i=1}^{k} \lambda_i \qquad \text{(for } k \neq 1\text{)}$$

or

$$0 \leq \lambda\xi \leq \lambda_1 \qquad \text{(for } k = 1\text{)}$$

then

$$\text{Prob} \left\{ \sum_{i=1}^{k-1} \lambda_i < \lambda\xi \leq \sum_{i=1}^{k} \lambda_i \right\}$$

$$= \text{Prob} \left\{ \frac{1}{\lambda} \sum_{i=1}^{k-1} \lambda_i < \xi \leq \frac{1}{\lambda} \sum_{i=1}^{k} \lambda_i \right\}$$

$$= \frac{1}{\lambda} \left(\sum_{i=1}^{k} \lambda_i - \sum_{i=1}^{k-1} \lambda_i \right)$$

$$= \lambda_k/\lambda \ .$$

which is the probability of going to the k-th state after present state is vacated. The generation of only one random number is required to determine the next state using this method.

C) Estimation of Reliability Indices from a State History

The steady state probability, frequency and the average duration of a state can be estimated from the total time durations in the state, total number of occurrences of the state and the total simulation time obtained from the state history as follows:

$$\text{probability} = \frac{\text{total time duration in the state}}{\text{total simulation time}}$$

$$\text{frequency} = \frac{\text{total number of occurrences in the state}}{\text{total simulation time}}$$

$$\text{average time duration} = \frac{\text{total time duration in the state}}{\text{total number of occurrences in the state}}.$$

The above equations also apply to capacity state reliability calculations.

The above procedure was usually repeated 25 times to obtain the mean value and the standard error of the mean value. Exact reliability values were obtained by solving the system transition matrix and are listed in Table 4.1. The results obtained by simulation are shown in Table 4.2. The total simulation time was 4,434 years. The total computer time required for this simulation was 1.14 minutes. The simulation was repeated with a longer simulation time and a reduction in standard error

Table 4.1: Steady State Reliability Indices for Three Phase Transformer Bank Consisting of Three Single Phase Transformers With A Spare Obtained by an Exact Analytical Method

State Values

State	Probability	Frequency (per year)	Average Duration (year)
1	0.9393253	0.7044940	1.3333333
2	0.0548742	0.6996455	0.0784314
3	0.0038337	0.7456496	0.0051414
4	0.0017148	0.0411556	0.0416667
5	0.0002521	0.0460041	0.0054795

Capacity Values

State	Probability	Frequency (per year)	Average Duration (year)
up	0.9941995	0.7456496	1.3333333
down	0.0058006	0.7456496	0.0077792

Table 4.2: Steady State Reliability Indices for the Three Phase Transformer Bank System Using a "Simulation Method." The Total Computer Time is 1.14 Minutes.

State Values

State	Probability	Frequency (per year)	Average Duration (year)	Standard Errors: Probability	Standard Errors: Frequency	Standard Errors: Average Duration
1	0.9387058	0.7136162	1.3233050	0.13868×10^{-2}	0.10948×10^{-1}	0.21488×10^{-1}
2	0.0555037	0.7097690	0.0781871	0.13376×10^{-2}	0.11408×10^{-1}	0.14056×10^{-2}
3	0.0037380	0.7559886	0.0049526	0.66702×10^{-4}	0.11702×10^{-1}	0.68469×10^{-4}
4	0.0018053	0.0424065	0.0440677	0.20243×10^{-3}	0.33523×10^{-2}	0.47631×10^{-2}
5	0.0002472	0.0464118	0.0053348	0.27204×10^{-4}	0.41086×10^{-2}	0.40051×10^{-3}

Capacity Values

State	Probability	Frequency (per year)	Average Duration (year)	Standard Errors: Probability	Standard Errors: Frequency	Standard Errors: Average Duration
up	0.9942095	0.7561808	1.3221764	0.23955×10^{-3}	0.11643×10^{-1}	0.20082×10^{-1}
down	0.0057905	0.7559886	0.0076460	0.23955×10^{-3}	0.11702×10^{-1}	0.26694×10^{-3}

was observed. The computer simulation process is shown in Figure 4.4

Example 2 X (SKIP TO PAGE 115)

In this example the three phase transformer bank with a spare is again considered, but the repair is assumed to be restricted. The repair on a unit is not started until all previously failed units have been repaired and installed (Figure 4.5). The repair and installation times are assumed to be non-exponentially distributed. The repair and installation rates are no longer constant but depend on the repair and installation times. The model is constructed for the general non-exponential case. Simulation is performed assuming the special Erlangian distribution for both repair and installation time durations. The characteristics of this special Erlangian distribution are discussed in Chapter 3. This distribution is used because the random variables associated with it can be easily generated and because accurate reliability evaluation can be done using the method of stages discussed in Chapter 3.

The following parameter values were used:

λ = 0.008 failures/year

M.R.T. = mean repair time = 182.5 days

M.I.T. = mean installation time = 0.5 days.

A) Generation of Repair and Installation Time Duration

A random variable associated with a special Erlangian distribution can be generated by summing up a number of random numbers obtained from an identical exponential distribution. If the number of stages are m and n, respectively, for the repair and installation processes, then the repair and installation times can be obtained as follows:

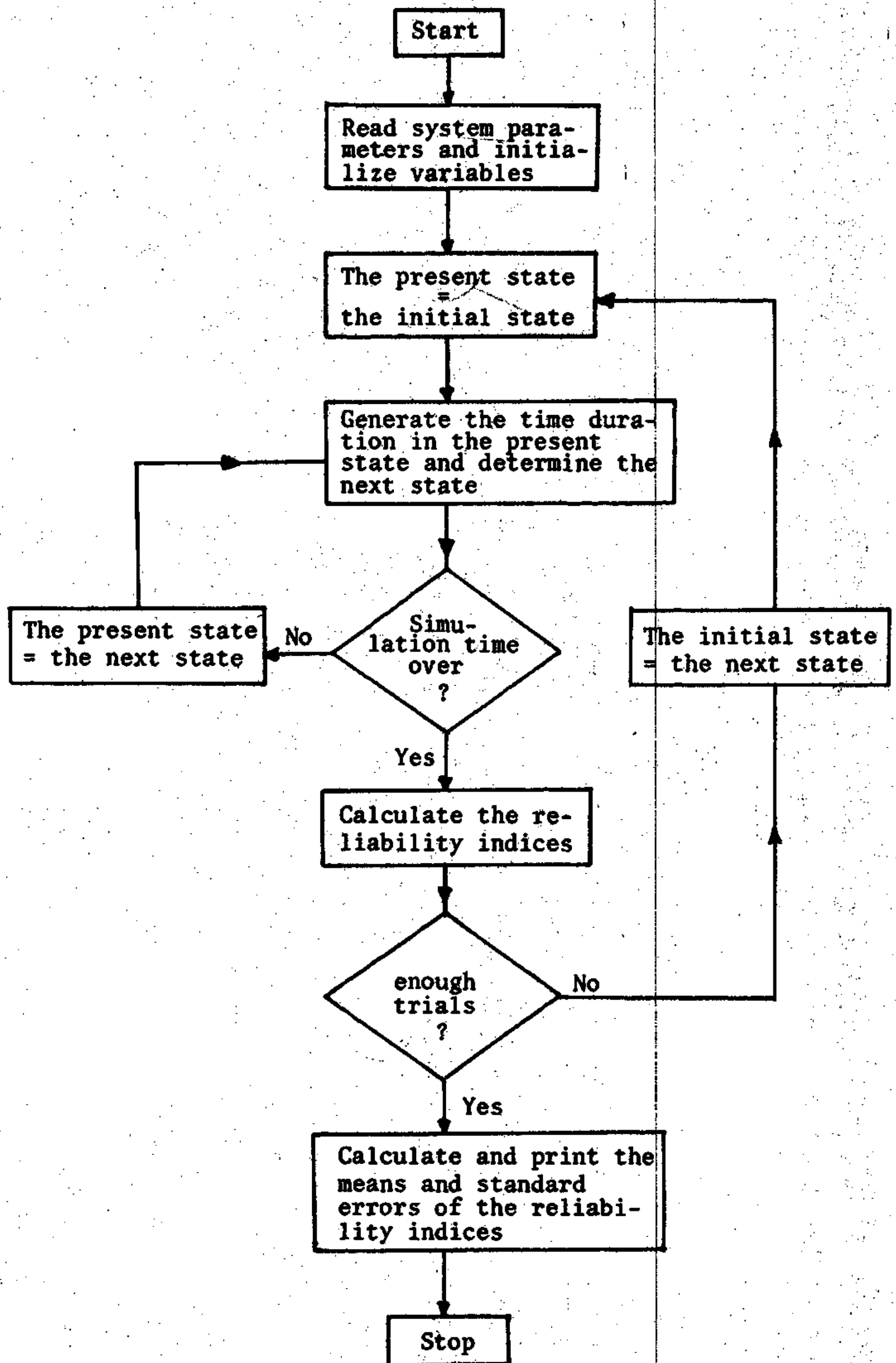

Figure 4.4: Flow Chart of the Simulation Program for Markovian Systems.

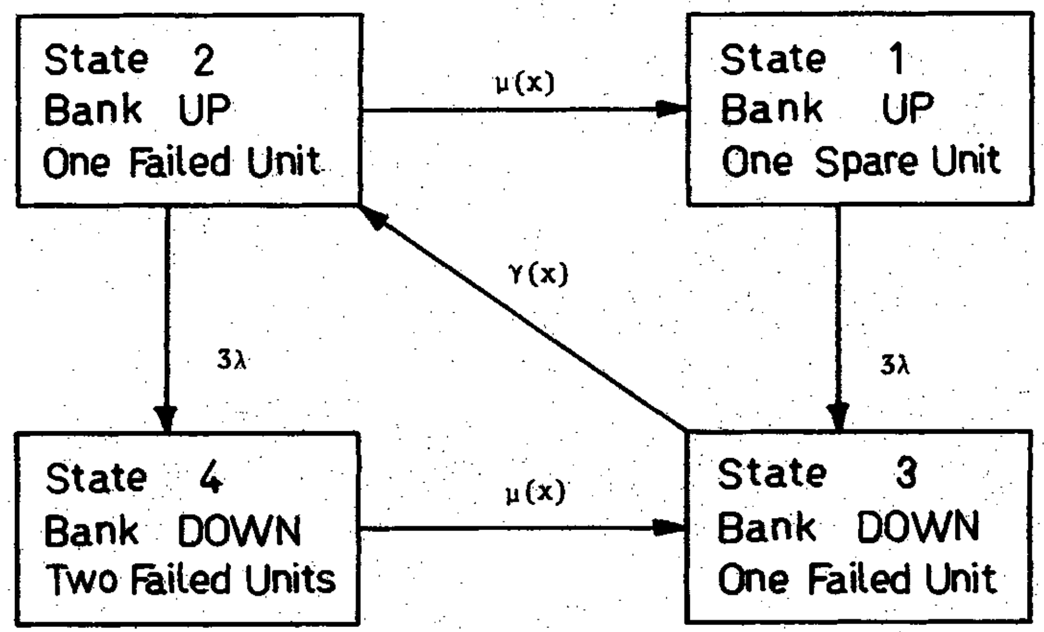

Figure 4.5: State Space Diagram of the 3 Phase Transformer Bank System Assuming Restricted Repair. The Repair and Installation Times are Non-exponentially Distributed.

Repair time

$$x = -\frac{M.R.T.}{m} \sum_{i=1}^{m} \log(\xi_i)$$

Installation time

$$x = -\frac{M.I.T.}{n} \sum_{i=1}^{n} \log(\xi_i)$$

where ξ_i's are uniformly distributed random numbers which have no correlation between each other.

B) Determination of the Next Event

The process is non-Markovian and the computer model construction of this example is more complicated than the previous case. The failure depends on the past history of the process. There exists three types of events possible in this case: failure, repair and installation. Times of occurrences of all future events must be generated and the event with the smallest time of occurrence becomes the next event. The computer process is shown in Figure 4.6.

The simulation results, assuming that the number of stages for repair and installation is three, are listed in Table 4.3, together with standard errors for these values. The example is accurately solved by the method of stages (Figure 4.7) and the results are shown in Table 4.4. The unavailabilities have been calculated by simulation with the number of stages varying from 1 to 5 and are listed in Table 4.5, together with the exact values obtained using the method of stages.

If the non-exponential distribution describing the repair or

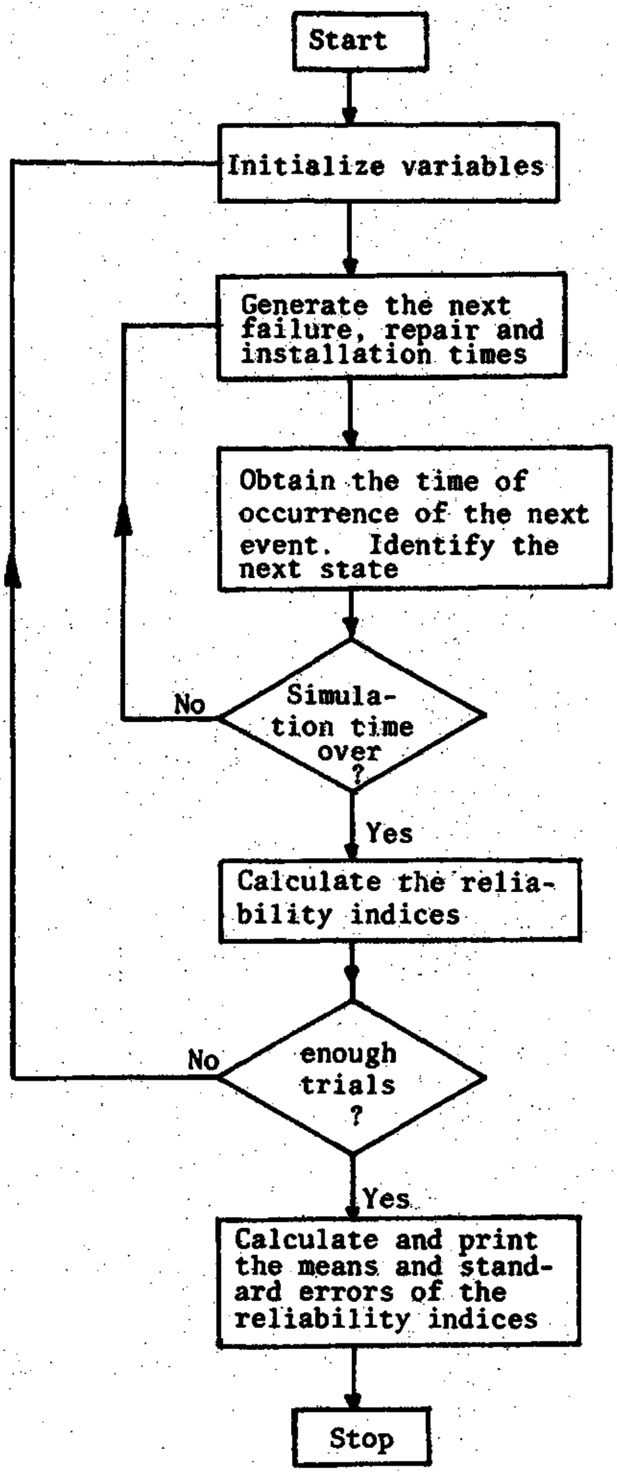

Figure 4.6: Flow Diagram of the Simulation Program for the Transformer Bank System with Non-exponential Repair and Installation Times.

Table 4.3: Steady State Reliability Indices for Three Phase Transformer Bank System Obtained by Simulation Assuming Repair Restriction and Special Erlangian Distribution for the Repair Period. Simulation Time = 50,000 Years.

State Values

				Standard Errors In		
State	Probability	Frequency (per year)	Average Duration (year)	Probability	Frequency	Average Duration
1	0.9880477	0.0237560	41.6134472	0.76213×10^{-4}	0.11012×10^{-3}	0.19740
2	0.0118200	0.0240264	0.4919673	0.71329×10^{-4}	0.11495×10^{-3}	0.19000×10^{-2}
3	0.0000330	0.0240264	0.0013743	0.15870×10^{-6}	0.11495×10^{-3}	0.29573×10^{-5}
4	0.0000993	0.0002904	0.3334816	0.94694×10^{-5}	0.16416×10^{-4}	0.18748×10^{-1}

Capacity Values

				Standard Errors In		
State	Probability	Frequency (per year)	Average Duration (year)	Probability	Frequency	Average Duration
up	0.9998677	0.0240464	41.6039469	0.95310×10^{-5}	0.11495×10^{-3}	0.20215
down	0.0001323	0.0240264	0.0054915	0.95310×10^{-5}	0.11495×10^{-3}	0.38448×10^{-3}

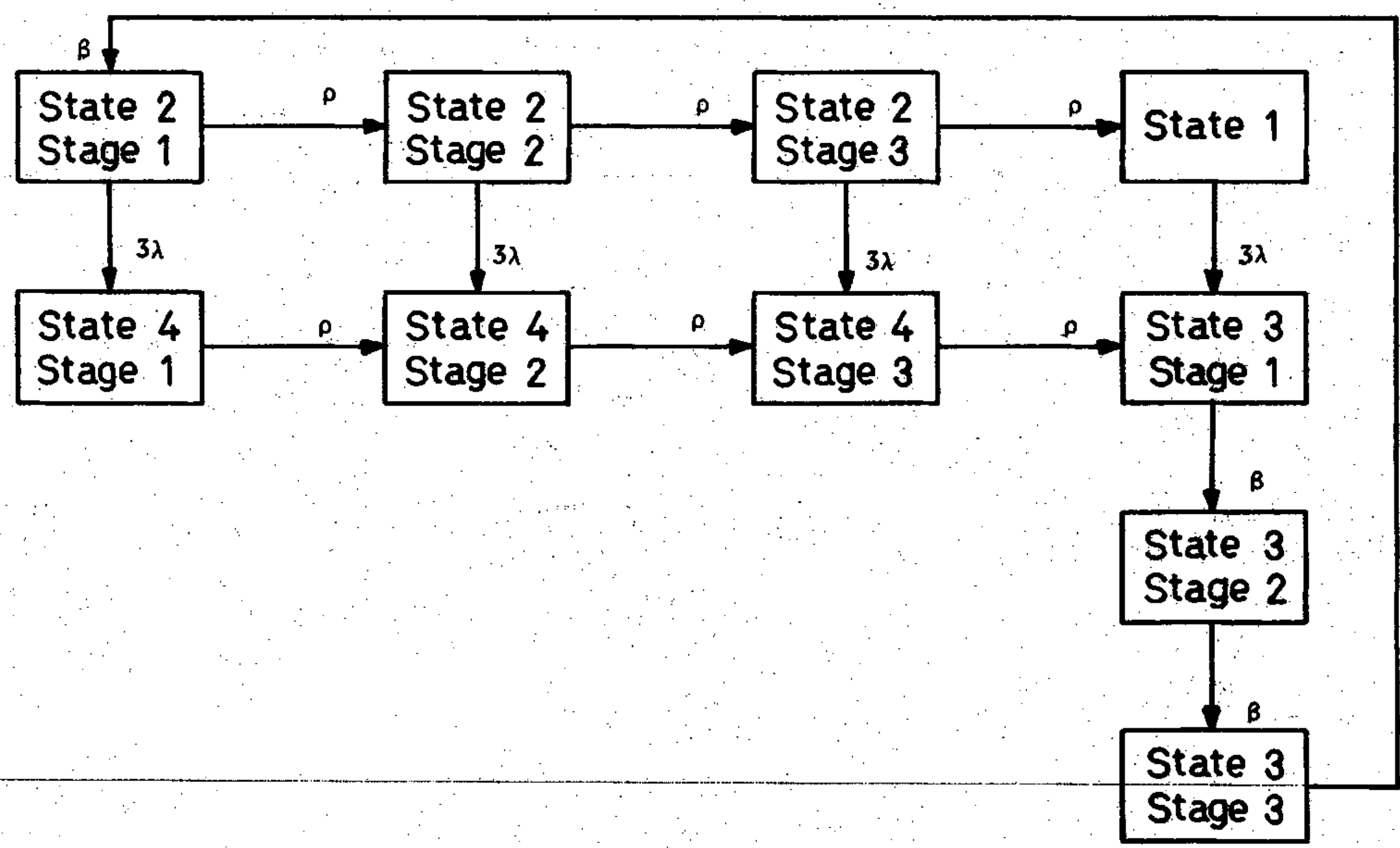

Figure 4.7: The State Transition Diagram of Three Phase Transformer Bank System Assuming the Special Erlangian Distribution for Repair and Installation Times.

Table 4.4: Steady State Reliability Indices for the Three Phase Transformer Bank System Obtained by the Analytical Method Assuming Repair Restriction and Special Erlangian Distribution for the Repair Period

State Values

State Number	Probability	Frequency (per year)	Average Duration (year)
1	0.9879687	0.0237112	41.6666663
2	0.0119031	0.0239969	0.4960265
3	0.0000329	0.0239969	0.0013699
4	0.0000954	0.0002857	0.3337769

Capacity Values

State	Probability	Frequency (per year)	Average Duration (year)
up	0.9998718	0.0239969	41.6666663
down	0.0001282	0.0239969	0.0053434

Table 4.5: Unavailabilities of the Three Phase Transformer Bank System Assuming Repair Restriction Obtained Using Both Analytical and Simulation Methods. The number of Stages for the Repair Process was Varied from 1 to 5.

Number of identical stages, "a", in the repair process	By Analytical Method	Unavailability By Simulation Method — Simulation Time: 2000 Years		50,000 Years		100,000 Years	
	Unavailability	Unavailability	Std. Error	Unavailability	Std. Error	Unavailability	Std. Error
1	0.0001751	0.0001437	0.3218×10^{-4}	0.0001865	0.1064×10^{-4}	0.0001841	0.7164×10^{-5}
2	0.0001400	0.0001266	0.4149×10^{-4}	0.0001397	0.0804×10^{-4}	0.0001342	0.5415×10^{-5}
3	0.0001282	0.0001574	0.4210×10^{-4}	0.0001323	0.0953×10^{-4}	0.0001322	0.5075×10^{-5}
4	0.0001223	0.0000800	0.1665×10^{-4}	0.0001222	0.0614×10^{-4}	0.0001290	0.4277×10^{-5}
5	0.0001187	0.0001438	0.3771×10^{-4}	0.0001091	0.0420×10^{-4}	0.0001234	0.4571×10^{-5}

installation process is not a special Erlangian distribution, the exact solution becomes more difficult. The same basic simulation program can, however, be used for reliability calculations. The changes required are in the subroutines for generation of the repair and installation times.

4.4 Application to the Short-term Reliability Evaluation

Example 3

The simulation approach was applied to the generation system example discussed in Chapter 3 (Figure 3.13). The up time is assumed to be exponentially distributed and the down time to be lognormally distributed. The following system parameter values were used:

mean up time = 1500 hours

μ = mean down time = 20 hours

σ = standard deviation of down time = 10 hours.

The probability of residing in the down state up to 24 hours in one hour intervals was obtained by simulation. Two different sampling methods were used for this example.

Method 1 Crude Monte Carlo Method

In this method the state history is simulated by generating the up and down times repeatedly until the total simulation is over 24 hours. The system is initially assumed to be in the up state. The number of occurrences of the down state is increased by one at time t, if the system exists in the down state at time t in any trial. If it is repeated many times, the down state probability can be obtained as follows:

$$P\ down\ (t) = \frac{\text{number of occurrences of down state at } t}{\text{total number of trials}}.$$

The mean and standard error of the result can be obtained by repeating the trial several times. The process is shown in Figure 4.8. (Next page) (P.117)

Generation of a Lognormally Distributed Random Variable

If the down time is lognormally distributed, it can be generated from a number of uniformly distributed random numbers. In this case the mean, ρ, and the standard deviation, α, of the normally distributed random variable can be obtained from the mean, μ, and the standard deviation, σ, of the lognormally distributed random variable by simple equations:

$$\rho = \log \mu - \tfrac{1}{2} \log [(\tfrac{\sigma}{\mu})^2 + 1]$$

$$\alpha^2 = \log [(\tfrac{\sigma}{\mu})^2 + 1].$$

The normally distributed random variable with this mean and standard deviation can be obtained by adding up N independent random numbers from an identical uniform distribution. When N is large, it approaches the normal distribution due to the Central Limit Theorem. N=12 is enough for practical purposes. The down time is then obtained by taking the antilogarithm of this normally distributed variable (reference 14).

The results obtained by this method are listed in Table 4.6.

Method 2 Stratified Sampling Method

A more efficient sampling method can be devised noting the fact

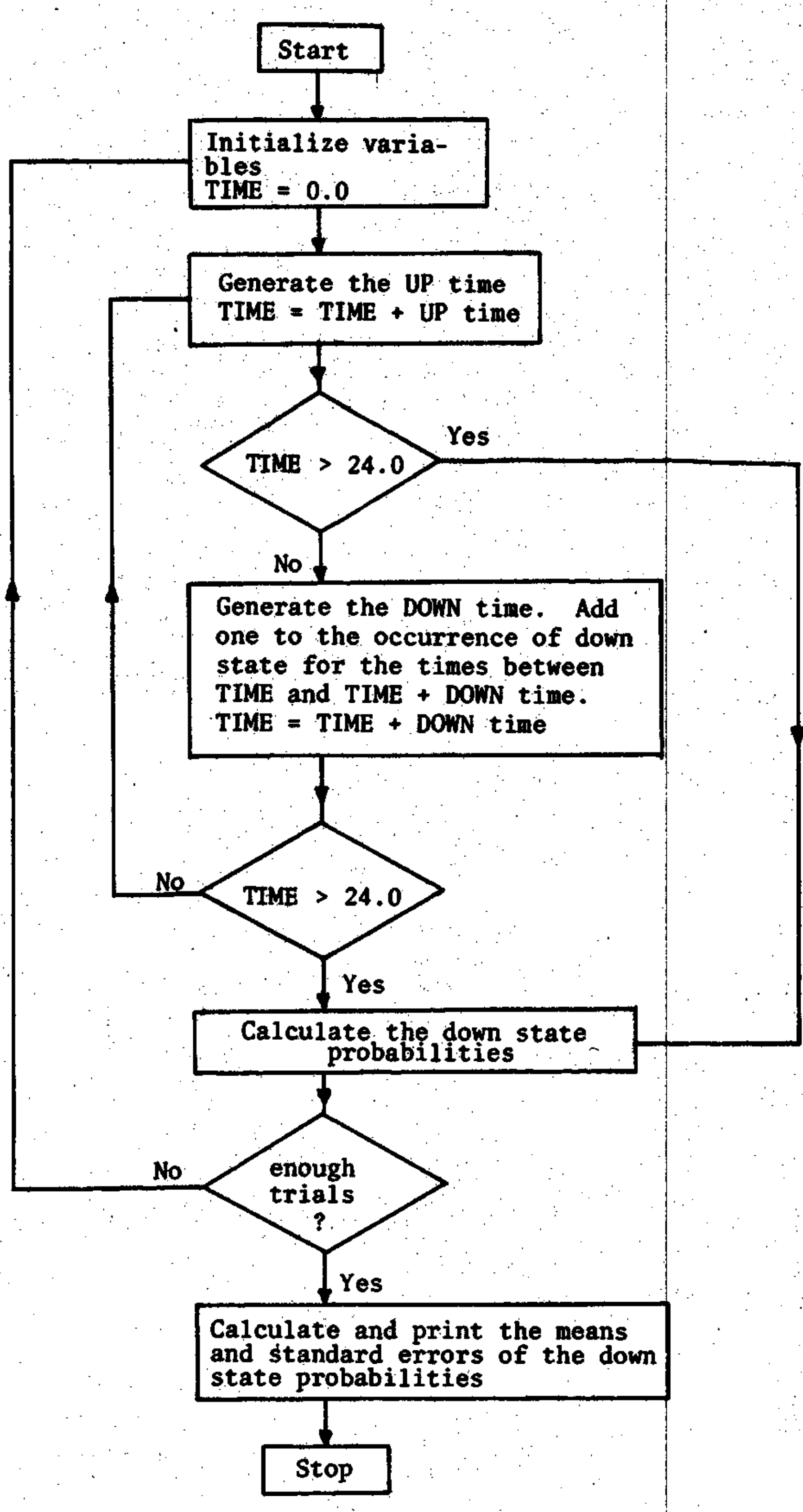

Figure 4.8: Flow Diagram of the Simulation Program for the Generation System.

Table 4.6: The Down State Probability of the Two State Generation System Model by Simulation Method

Time	Computer time = 13.16 secs		Computer time = 28.90 secs	
(hours)	Probability	Standard Error	Probability	Standard Error
1	0.000610	0.000166	0.000741	0.000121
2	0.000949	0.000221	0.001452	0.000179
3	0.001966	0.000348	0.002370	0.000214
4	0.002576	0.000439	0.003022	0.000245
5	0.003593	0.000521	0.003674	0.000272
6	0.004339	0.000642	0.004563	0.000328
7	0.005085	0.000706	0.005067	0.000403
8	0.006034	0.000733	0.005778	0.000415
9	0.006847	0.000755	0.006696	0.000400
10	0.007254	0.000723	0.007141	0.000375
11	0.008136	0.000739	0.007615	0.000431
12	0.008271	0.000721	0.007881	0.000434
13	0.008746	0.000683	0.008504	0.000410
14	0.009085	0.000698	0.008948	0.000385
15	0.009559	0.000738	0.009304	0.000473
16	0.009898	0.000762	0.009541	0.000475
17	0.010508	0.000813	0.010163	0.000516
18	0.011119	0.000759	0.010844	0.000534
19	0.011458	0.000772	0.011141	0.000567
20	0.011390	0.000716	0.011200	0.000563
21	0.011458	0.000728	0.011170	0.000580
22	0.011186	0.000752	0.011230	0.000627
23	0.011254	0.000684	0.011496	0.000617
24	0.011186	0.000706	0.011437	0.000596

that the probability of leaving the up state within 24 hours is very small. The down state probability at time t can be considered as a function of the initial up time duration which is in turn a function of a uniformly distributed random variable $\xi \in (0, 1)$. Therefore, the following relationship holds:

$$\xi \rightarrow \text{initial up time} \rightarrow P\text{ down }(t)$$

or

$$P\text{ down }(t) = f(\xi).$$

The initial up time is broken into two ranges:

a) time ranging from 0 to 24 hours

b) time greater than 24 hours.

Case a) occurs when $1 > \xi > e^{-24\lambda}$ and case b) when $0 < \xi < e^{-24\lambda}$.

Applying the Monte Carlo method separately to each case, the down state probability can be obtained by (reference 16):

$$\theta = \sum_{i=1}^{n_1} (1 - e^{-24\lambda}) \frac{1}{n_1} f\{e^{-24\lambda} - (1 - e^{-24\lambda}) \xi_{i\,1}\}$$

$$+ \sum_{i=1}^{n_2} (e^{-24\lambda}) \frac{1}{n_2} f(e^{-24\lambda} \xi_{i\,2})$$

where n_1 and n_2 are the number of trials for case a) and case b), respectively.

The down state probability at time t between 0 and 24 hours will be zero for case b) and the above equation can be reduced to:

$$\theta = \sum_{i=1}^{n} (1 - e^{-24\lambda}) \frac{1}{n} f\{e^{-24\lambda} - (1 - e^{-24\lambda}) \xi_i\}$$

where n is the total number of trials.

The down state probabilities obtained by this method are listed in Table 4.7. The efficiency gain by this sampling method over the Crude Monte Carlo method can be approximately calculated by comparing the standard errors and computation times. The efficiency gain using the standard error values at t=24 hours is:

$$\left(\frac{0.000706}{0.000132}\right)^2 \cdot \frac{13.16}{12.87} \approx 50 .$$

This method is approximately 50 times as efficient as the Crude Monte Carlo method. The results are plotted in Figure 4.9. The solid line is the values obtained in Chapter 3.

4.5 Conclusion

The simulation method has been applied to several example systems in this chapter for both long-term and short-term reliability evaluations. The results, together with the standard errors, have been presented. The problems were solved by other exact reliability techniques whenever possible and the results presented to compare these methods and the simulation method. The results generally show relatively good agreement.

There can be more than one simulation method for a given problem. The simulation models constructed in this chapter are simple examples of possible Monte Carlo simulations. The efficiency of a simulation may be improved by using a better sampling technique. There exists no general

Table 4.7: The Down State Probability of the Two State Generation System Model by Simulation Method Using Stratified Sampling

Computer time = 12.87 secs

Time	Probability	Standard Error
1	0.000756	0.000072
2	0.001371	0.000085
3	0.002019	0.000107
4	0.002686	0.000117
5	0.003352	0.000129
6	0.003994	0.000145
7	0.004641	0.000170
8	0.005232	0.000178
9	0.005797	0.000196
10	0.006292	0.000178
11	0.006933	0.000162
12	0.007536	0.000160
13	0.008082	0.000170
14	0.008628	0.000152
15	0.009086	0.000147
16	0.009498	0.000150
17	0.009911	0.000149
18	0.010336	0.000125
19	0.010622	0.000138
20	0.011003	0.000125
21	0.011301	0.000106
22	0.011473	0.000144
23	0.011682	0.000156
24	0.011746	0.000132

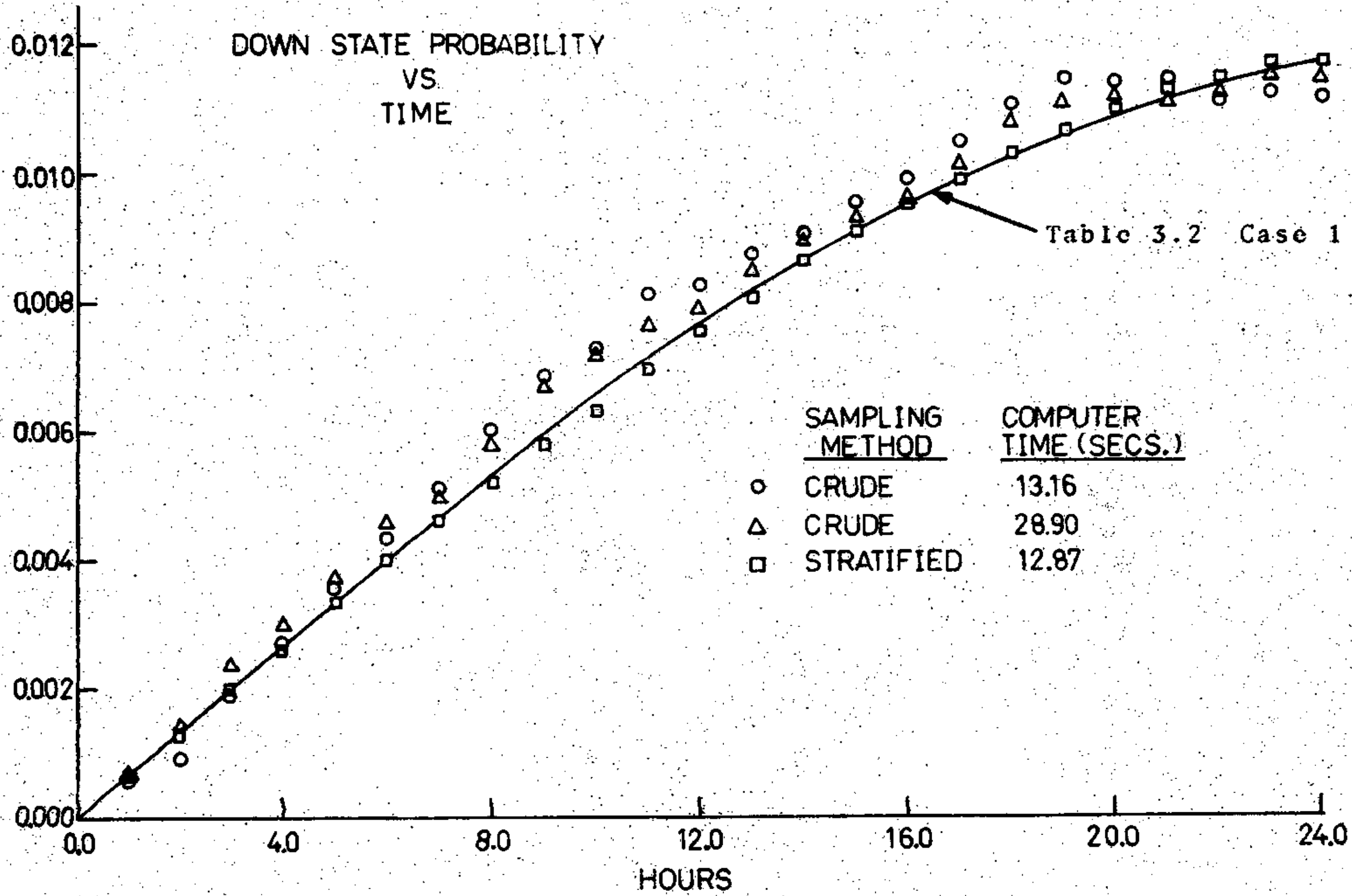

Figure 4.9: The Down State Probability of the Generation System Obtained by Simulation Methods.

theory about which sampling technique will give the best overall efficiency. Accelerated methods usually require more time in constructing computer programs and, therefore, can be less preferable than straightforward Crude Monte Carlo simulations, particularly in a single study case.

The concept of simulation efficiency is very useful in comparing different simulation models for a given problem. The standard error in a result is not only a measure of the precision in the result but also a necessary value to obtain the efficiency of the simulation method ~~used~~.

The Monte Carlo method is well-suited for a problem composed of deterministic and stochastic processes. The behaviour of the system components and the rules or policies of the system can easily be incorporated into a simulation model. Analytical methods sometimes require very complicated mathematical techniques to solve for reliability indices. The simulation method, therefore, becomes more attractive than the analytical method in some cases. For certain problems simulation can be the only method to obtain reliability indices.

5. CONCLUSIONS

As noted in the introduction to this thesis, the techniques for reliability evaluation in power system are limited in their application to realistic problems. Many simplifying assumptions must often be made for a system so that calculations of desired reliability indices by existing techniques are economically feasible. Mathematical reliability models are, therefore, approximate representations of the physical system characteristics relevant to reliability. These models do not always adequately represent the given systems. In many cases, there are sufficient mathematical concepts available that can be applied to better system modelling. Appropriate computational techniques which are essential for utilizing these mathematical concepts may not exist. This thesis presents developments in three different techniques for reliability evaluation: a method of large Markovian system modelling, a method for non-Markovian models (the method of stages), and the simulation method. These techniques may not be required if the system can be reasonably well represented by simple mathematical models. Those methods, however, may provide more freedom in making the required assumptions for many power system models.

One of the most widely applied mathematical theories in power system reliability modelling is the concept of Markov processes. Markovian models provide a reasonably good representation for many systems and the reliability predictions made from the models are relatively easy to obtain. One major disadvantage is that the model size increases very rapidly as the system size becomes larger. In most cases of large system modelling, the dependencies between the system components are assumed to be negligible and the problem becomes one of modelling a number of small systems instead of a large one. This assumption cannot always be applied and construction or solution of the large model becomes difficult. A method of modelling a large system has been developed and presented in Chapter 2 in an attempt to reduce this difficulty.

This technique has been applied to a heat transport pump system. Reliability indices are calculated under three different pump failure mode assumptions. The effect of different assumptions on the reliability indices was examined. Temporary failures have a relatively small effect on system availability. The effect can, however, be quite significant when there is a relatively large number of spares such that the unavailability due to permanent pump outages is comparable to that due to temporary pump outages. The inclusion of temporary failures has a considerable effect on the frequency of system failure. If the frequency index is an important parameter, then the temporary failures must be included in the model. The importance of the frequency index depends upon the cost associated with system failure in addition to the loss of generated energy measured by the unavailability indices. The plant availability can be improved by the provision of spares, but the frequency of failure is not reduced by their provision. Classification of permanent failures into two groups of short and long repair times has shown only a small effect on both availabilities and frequencies at steady state. This assumption does not, therefore, appear to justify the increasing complexity in modelling.

Large state space models such as those shown in Chapter 2 are not really necessary. The diagrams or tables are useful only when they aid in understanding the system. The resulting state space models are extremely huge and complex and, therefore, they may not have much meaning in total presentation. A more important aspect is the various contingency characteristics of the system components or overall system, the distributions associated with those contingencies, decision policies for various possible occasions and other assumptions and system characteristics. The state space model can be obtained by a routine process from those facts which can be taken over by a computer as shown in Chapter 2.

The method of stages is a technique of evaluating reliabilities in those systems which cannot be well represented by Markovian models. This technique has been presented and applied to an example system in Chapter 3. This method approximates non-Markovian models by Markovian models. The method is general and practical. The advantages of this method are as follows:

1. The final model becomes Markovian, which can be relatively easily solved either for long-term or short-term studies.

2. The approximation can be performed first for each non-exponential distribution in the non-Markovian model and then the entire model can be easily obtained from these approximations. The overall system behaviour need not be considered in the approximation process.

3. Method of stages can be applied for either accurate or crude approximation of a non-Markovian model. The degree of accuracy shown in the results of the example system in Chapter 3 may not be necessary in many practical systems. Simpler stage combinations can be used with less accuracy. Even a very crude approximation by this method will show better results than that obtained assuming the system to be Markovian.

Only a brief discussion is given in this thesis for the simulation approach. This is a very powerful technique that can be applied virtually to any problem and the desired accuracy in the result can be theoretically achieved if enough trials are made. It is not practical to discuss this technique in general terms and it is mainly explained by application to actual problems. The efficiency of a simulation method is as important as that of other analytical techniques. The simulation method becomes practical only if analytical methods fail because simulation techniques have lower efficiencies than a direct analytical method. Accelerated simulation methods can be used to improve the efficiency, but it requires more effort to develop these methods than the simple simulation approach. This improvement can be quantitively examined by comparing the efficiencies of different simulation methods.

If there exists more than one technique for a given reliability problem, the consideration of efficiency must be the main criterion for the choice of a particular technique over other techniques. The efficiency is the measure of the output accuracy with respect to the total efforts and expenses including human labour and computer time. The techniques presented in this thesis are only some possible methods of reliability evaluation and these methods must be compared with other existing methods before applying one of them to a particular problem.

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APPENDIX 1.

CALCULATION OF THE NUMBER OF STATES OF HT PUMP MODELS

1. Single Failure Mode

The number of states for a unit consisting of four pumps is six. If there are n units in the plant, the number of states in the site is $\binom{n+5}{5}$, where

$$\binom{a}{b} = \frac{a\,(a-1)\,\cdots\,(a-b+1)\;(a-b)!}{b!\;(a-b)!} = \frac{a!}{b!\,(a-b)!}$$

If there are m common spares, the number of states required to describe the whole plant operation will become:

$$\binom{n+5}{5} + m\binom{n+2}{2} .$$

The numbers of states are calculated from the above expression and are listed in Table A1.

2. Mixed Failure Mode

The number of states for a unit now becomes 10. The total number of states for n units and m spares can be given as follows:

$$N = \binom{n+9}{9} + m\binom{n+5}{5} .$$

The numbers of states are listed in Table A2.

3. Two Failure Modes

The numbers of states are given in Table A3.

Table A1: Numbers of States in the HT Pump Models Assuming a Single Failure Mode

Number of Units (n)	Number of Spares (m)				
	0	1	2	3	4
1	6	9	12	15	18
2	21	27	33	39	45
3	56	66	76	86	96
4	126	141	156	171	186

Table A2: Numbers of States in the HT Pump Models Assuming the Mixed Failure Modes

Number of Units (n)	Number of Spares (m)				
	0	1	2	3	4
1	10	16	22	28	34
2	55	76	97	118	139
3	220	276	332	388	444
4	715	841	967	1093	1219

Table A3: Numbers of States in the HT Pump Models Assuming the Two Permanent Failure Modes

Number of Units (n)	Number of Spares (m)				
	0	1	2	3	4
1	10	19	31	46	64
2	49	76	109	148	193
3	168	234	310	396	492
4	462	603	759	930	1116

APPENDIX 2.

A COMPUTER METHOD FOR REPAIRABLE SYSTEM STATE SPACE MODEL CONSTRUCTION

A general method for state space construction has been developed and was used for the problems encountered in Chapter 2. It can also be used for other similar problems. The method is particularly suitable for construction of state space models with a large number of states. A model with 1219 states has been developed by the method.

A computer program for reliability calculation can be combined with the computer program for state space construction. If this is done, the problem of input to the computer of the transitional matrix of the Markovian process disappears. Considerable effort is required to input a transition matrix if the number of states is very large. The computer method also eliminates ordinary human errors in state model construction and transition rate input procedures.

An adequate method of representing states with a set of integer parameters must be established prior to computer state space model construction. There can be several possible methods. The methods utilized in this thesis for representing the states used in the heat transfer pump system were explained in Chapter 2.

The state model construction of a process by the computer method requires the clear understanding of the process. The following are some of the basic properties in a repairable system process:

(a) a component in operation eventually fails,

(b) a failed component can be repaired and put into operation,

(c) the number of components in the system does not increase or decrease.

All possible situations requiring a decision must be anticipated for consistant state model construction. Changes in state parameters due to each event are incorporated into the main program.

If an event occurs to a system existing in a given state, the state changes into another state. States can be generated from a given condition assuming all possible events happen to the condition. States are generated from an initial state and the resultant states are recorded. Repeating the process again leads to other states. This procedure is continued until the whole state space model is constructed.

New states generated during the procedure are numbered and stored in order of generation. Each state in the list is selected one by one and all possible events in the state are assumed to happen so that states can be generated sequentially. State generation stops when the total number of states generated, including the initial state, is equal to the total number of states which have been exposed to all contingencies. The complete process is shown in Figure A1.

The outputs of the program are: (a) listings and descriptions of states generated and; (b) the list of all transition rates between the states. The input data required for the method are: (a) the number of parameters needed to represent a state; (b) the number of possible type of events in the system; (c) the rates associated with those events;

(d) the parameter values of the initial state. Taking state space model construction of four units of heat transfer pump systems, for example, the input data for one spare assuming single failure mode are: (a) 8, the number of state parameters; (b) 3, the number of possible events (failure, repair, installation); (c) 0.6 for failure rate, 35.04 for repair rate, 292.0 for installation rate; (d) 4, 0, 0, 0, 0, 0, 0, 1 for initial state. All four units are producing full power and a spare is stored in the warehouse in the initial state. Any other state can be taken as initial state.

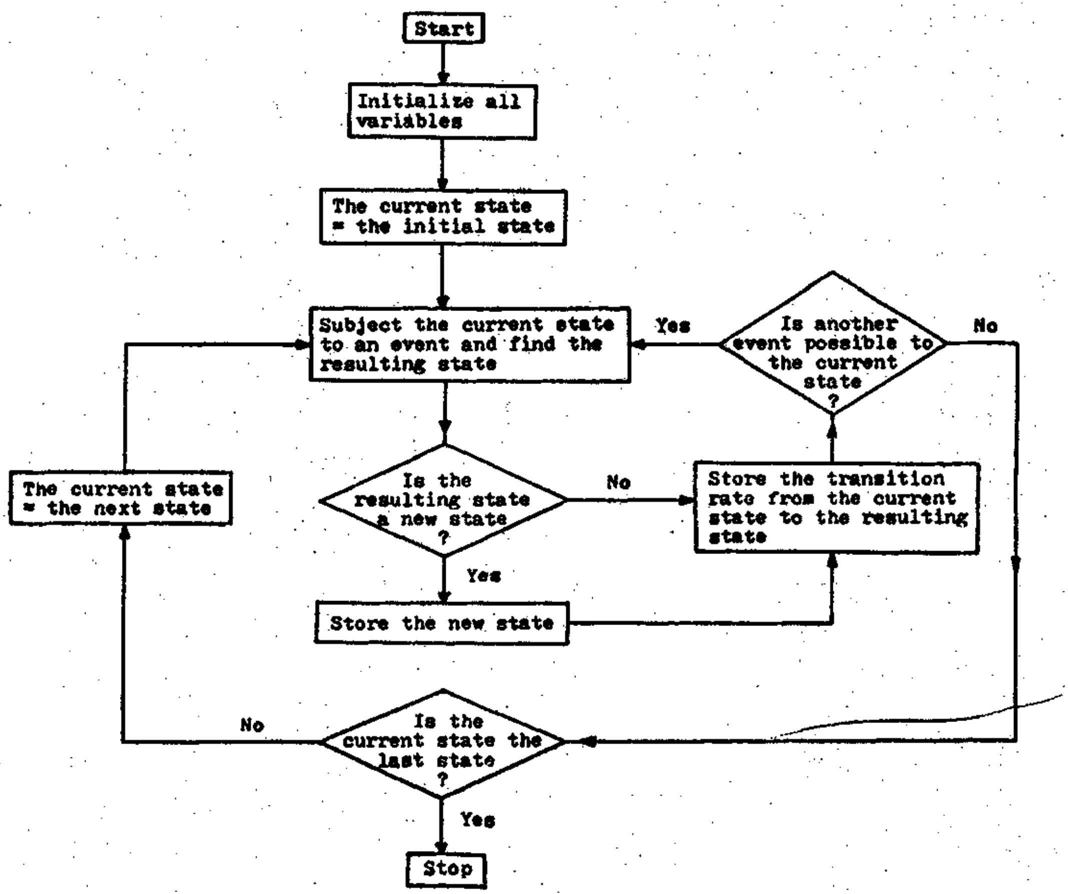

Figure A1: Flow Diagram for the Computer Construction of State Space Models.

APPENDIX 3.

MOMENT CALCULATION FOR SOME STAGE COMBINATIONS

The rth moment of the random variable is by definition the expectation of η^r; $M^r = E(\eta^r)$. This concept is utilized in the moment matching technique described in Chapter 3. Another quantity called rth central moment is often used as a measure of distribution characteristics. The rth central moment of η is defined as $E\{(\eta-\mu)^r\}$ where $\mu = E(\eta)$. Calculation method of central moments using Laplace is also presented in this appendix.

1. rth Moment Calculation

(a) A series of identical stages

The Laplace transform of its p.d.f. is given by:

$$\bar{f}(s) = \left(\frac{\rho}{s+\rho}\right)^a \quad \text{or} \quad (s+\rho)^a \; \bar{f}(s) = \rho^a .$$

Differentiating both sides with respect to s,

$$(s+\rho)\,\bar{f}'(s) + a\,\bar{f}(s) = 0. \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Differentiating (r-1) times w.r.t. s,

$$(s+\rho)\,\bar{f}^{(r)}(s) + (a+r-1)\,\bar{f}^{(r-1)}(s) = 0. \qquad \ldots\ldots\ldots (2)$$

Introducing s=0 to equations (1) and (2),

$$\rho\ \bar{f}'(0) + a = 0 \qquad \ldots\ldots\ldots\ldots (1)'$$

and
$$\rho\ \bar{f}^{(r)}(0) + (a + r - 1)\ \bar{f}^{(r-1)}(0) = 0. \qquad \ldots\ldots\ldots\ldots (2)'$$

Deducing from (1)' and (2)',

$$M^r = (-1)^r\ \bar{f}^{(r)}(0)$$

$$= \frac{1}{\rho^r} \prod_{k=1}^{r} (a + k - 1).$$

(b) Two series stages in parallel

The Laplace transform of its p.d.f. is given by:

$$\bar{f}(s) = \omega_1 \left(\frac{\rho_1}{s + \rho_1}\right)^{a_1} + \omega_2 \left(\frac{\rho_2}{s + \rho_2}\right)^{a_2} .$$

Let $\bar{f}_1(s) = \left(\frac{\rho_1}{s + \rho_1}\right)^{a_1}$ and $\bar{f}_2(s) = \left(\frac{\rho_2}{s + \rho_2}\right)^{a_2}$,

then
$$M^r = (-1)^r\ \bar{f}^{(r)}(0)$$

$$= (-1)^r\ \omega_1 \bar{f}_1{}^{(r)}(0) + (-1)^r\ \omega_2\ \bar{f}_2{}^{(r)}(0) .$$

Applying the result obtained in the previous combination to obtain $\bar{f}_1{}^{(r)}(0)$ and $\bar{f}_2{}^{(r)}(0)$:

$$M^r = \frac{\omega_1}{\rho_1^{\,r}} \prod_{k=1}^{r} (a_1 + k - 1) + \frac{\omega_2}{\rho_2^{\,r}} \prod_{k=1}^{r} (a_2 + k - 1)$$

(c) Series stages in series with a distinctive stage

The Laplace transform of its p.d.f. is given by:

$$\bar{f}(s) = \left(\frac{\rho}{s + \rho}\right)^a \left(\frac{\rho_1}{s + \rho_1}\right)$$

or $$(s + \rho)^a (s + \rho_1) \bar{f}(s) = \rho_1 \rho^a .$$

Differentiating both sides,

$$(s + \rho) (s + \rho_1) \bar{f}^1(s) + \{a (s + \rho_1) + (s + \rho)\} \bar{f}(s) = 0.$$

Differentiating (r-1) times,

$$(s + \rho) (s + \rho_1) \bar{f}^{(r)}(s) + \{r(s + \rho) + (a + r - 1) (s + \rho_1)\} \bar{f}^{(r-1)}(s)$$

$$+ (r - 1) (a + r - 1) \bar{f}^{(r-2)}(s) = 0.$$

Introducing s=0 and $\bar{f}^{(r)}(0) = (-1)^r M^r$,

$$\rho\rho_1 M^r - \{r\rho + (a + r - 1)\rho_1\} M^{r-1} + (r-1) (a + r - 1)M^{r-2} = 0.$$

The rth moment cannot be calculated individually like previous cases, but r simultaneous linear equations can be set up to obtain the first r moments. They are:

$$\rho\rho_1 M^1 = a\rho_1 + \rho \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

$$\rho\rho_1 M^2 - \{2\rho + (a + 1)\rho_1\} M^1 = -(a + 1) \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

- - - - -

$$\rho\rho_1 M^r - \{r\rho + (a + r - 1)\rho_1\} M^{r-1} + (r - 1)(a + r - 1) M^{r-2} = 0$$

$$\ldots\ldots\ldots (r)$$

The above equations can be expressed in matrix and vector form,

$$A_1 M = B_1$$

where $A_1 = [a_{ij}]$ is the coefficient matrix such that

$$a_{ij} = 0 \text{ if } j>i \text{ or } j<i-2,$$

$$a_{ij} = \rho\rho_1 \text{ if } i=j,$$

$$a_{ij} = -\{i\rho + (a + i - 1)\rho_1\} \text{ if } j = i-1,$$

$$a_{ij} = (i - 1)(a + i - 1) \text{ if } j = i-2.$$

The moment vector M and the coefficient vector B are as follows:

$$M = [M^1 \; M^2 \; --- \; M^r]^t$$

$$B_1 = [\rho + a\rho_1 \quad -(a + 1) \quad 0 \quad --- \quad 0]^t.$$

(d) Series stages in series with two parallel stages

This combination is equivalent to two "series stages in series with a distinctive stage" in parallel (Figure 3.13b). The moment vector can be written as:

$$M = \omega_1 M_1 + \omega_2 M_2$$

where M_1 and M_2 are moment vectors for the two "series stages in series with a distinctive stage" combinations. M_1 and M_2 can be obtained by applying the result presented in the last section and solving for associated equations.

2. rth Central Moment Calculation

The rth central moment μ_r can be obtained by:

$$\mu_r = (-1)^r \frac{d^r}{ds^r} \{e^{\mu s} \bar{f}(s)\} \Big|_{s=0}$$

The probability density function associated with a series of identical stages has the following Laplace transform:

$$\bar{f}(s) = \left(\frac{\rho}{s + \rho}\right)^a .$$

Let $\bar{g}(s) = e^{\mu s} \bar{f}(s)$, then

$$(s + \rho)^a \bar{g}(s) = \rho^a e^{\mu s} .$$

Differentiating both sides,

$$(s + \rho) \bar{g}'(s) - \{\mu(s + \rho) - a\} \bar{g}(s) = 0. \qquad \ldots\ldots\ldots (1)$$

Differentiating the above equation again with respect to s,

$$(s + \rho) \bar{g}''(s) + \{a + 1 - \mu(s + \rho)\} \bar{g}'(s) - \mu \bar{g}(s) = 0 \qquad \ldots\ldots (2)$$

Differentiating both sides (r-2) times,

$$(s + \rho) \bar{g}^{(r)}(s) + \{a + r - 1 - \mu(s + \rho)\} \bar{g}^{(r-1)}(s) - \mu(r - 1) \bar{g}^{(r-2)}(s)$$

$$= 0. \qquad \ldots\ldots\ldots (3)$$

Introducing s=0 and then substituting $\mu = a/\rho$ and $\mu_r = (-1)^r \bar{g}^{(r)}(0)$ into the above equations,

$$\mu_1 = 0 \qquad \ldots\ldots\ldots\ldots\ldots (1)'$$

$$\rho\mu_2 = \mu \qquad \ldots\ldots\ldots\ldots\ldots (2)'$$

$$\rho\mu_r - (r-1)\mu_{r-1} + \mu(r-1)\mu_{r-2} = 0 \qquad (r \geq 3) \qquad \ldots\ldots\ldots\ldots (3)'$$

From the equations $(1)'$, $(2)'$ and $(3)'$, the first r central moments can be obtained. The second central moment is known as the variance. In this case the variance can be obtained from $(2)'$: the variance $\mu_2 = \mu/\rho$. The standard deviation is defined by:

$$\sigma = \sqrt{\mu_2} \quad ,$$

which is in this case

$$\sigma = \sqrt{2/\rho} \; .$$

The same procedure can be applied to the other stage combinations.

APPENDIX 4.

CALCULATION OF THE ELEMENTS OF THE JACOBIAN MATRIX OF ϕ

(a) Two Series Stages in Parallel

Assuming a_1 and a_2, the remaining three parameters ρ_1, ρ_2, and ω_1 can be calculated by matching the first three moments. The jth element of the vector ϕ is:

$$\phi_j = \frac{\omega_1}{\rho_1^{\,j}} \prod_{k=1}^{j} (a_1 + k - 1) + \frac{\omega_2}{\rho_2^{\,j}} \prod_{k=1}^{j} (a_2 + k - 1) - m^j .$$

The elements of the jth column of the Jacobian matrix of ϕ can be obtained by partially differentiating ϕ_j with respect to the parameters ρ_1, ρ_2, ω_1:

$$\frac{\partial \phi_j}{\partial \rho_1} = -\frac{j\omega_1}{\rho_1^{\,j+1}} \prod_{k=1}^{j} (a_1 + k - 1)$$

$$\frac{\partial \phi_j}{\partial \rho_2} = -\frac{j\omega_2}{\rho_2^{\,j+1}} \prod_{k=1}^{j} (a_2 + k - 1)$$

$$\frac{\partial \phi_j}{\partial \omega_1} = \frac{1}{\rho_1^{\,j}} \prod_{k=1}^{j} (a_1 + k - 1) - \frac{1}{\rho_2^{\,j}} \prod_{k=1}^{j} (a_2 + k - 1).$$

(b) Series Stages in Series with Two Parallel Stages

If the unknown parameters are ρ_1, ρ_2, ρ and ω_1, the parameters can be calculated by matching the first four moments. The combination consists of two "series stages in series with distinctive stage" combinations. Moment calculation technique as shown in the Appendix 3.1(c) can be applied to both combinations and following equations can be set up:

$$A_1M_1 = B_1 \qquad \text{and} \qquad A_2M_2 = B_2.$$

The two equations are equivalent to $A_1(\omega_1M_1) = \omega_1B_1$ and $A_2(\omega_2M_2) = \omega_2B_2$. Let $M_a = \omega_1M_1$, $M_b = \omega_2M_2$, $B_a = \omega_1B_1$ and $B_b = \omega_2B_2$; then the equations become $A_1M_a = B_a$ and $A_2M_b = B_b$.

The moment vector of the entire combination is given by $M = \omega_1M_1 + \omega_2M_2 = M_a + M_b$ according to the Appendix 3.1(d). The Jacobian matrix of ϕ at X_o therefore, becomes $\phi'(X_o) = M_a'(X_o) + M_b'(X_o)$. $M_a'(X_o)$ is the Jacobian matrix of M_a at X_o and can be obtained by partially differentiating $A_1M_a = B_a$ w.r.t. ρ, ρ_1, ρ_2, ω_1 and solving for the partial derivatives as follows:

$$A_1 \frac{\partial M_a}{\partial \rho} + \frac{\partial A_1}{\partial \rho} M_a = \frac{\partial B_a}{\partial \rho}$$

$$A_1 \frac{\partial M_a}{\partial \rho_1} + \frac{\partial A_1}{\partial \rho_1} M_a = \frac{\partial B_a}{\partial \rho_1}$$

$$A_1 \frac{\partial M_a}{\partial \rho_2} + \frac{\partial A_1}{\partial \rho_2} M_a = \frac{\partial B_a}{\partial \rho_2}$$

$$A_1 \frac{\partial M_a}{\partial \omega_1} + \frac{\partial A_2}{\partial \omega_1} M_a = \frac{\partial B_a}{\partial \omega_1} .$$

The above equation can be expressed by:

$$A_1 M_a' = C_1 ;$$

where C_1 is a 4x4 matrix such that the first column of C_1 is

$$\frac{\partial B_a}{\partial \rho} - \frac{\partial A_1}{\partial \rho} M_a,$$

the second $\quad \dfrac{\partial B_a}{\partial \rho_1} - \dfrac{\partial A_1}{\partial \rho_1} M_a,$

the third $\quad \dfrac{\partial B_a}{\partial \rho_2} - \dfrac{\partial A_1}{\partial \rho_2} M_a,$

the fourth $\quad \dfrac{\partial B_a}{\partial \omega_1} - \dfrac{\partial A_1}{\partial \omega_1} M_a.$

The elements of C_1 are obtained from the known B_a, A_1, M_a:

$$C_1 = \begin{pmatrix} -\rho_1 M_1^{\ 1} + \omega_1 & -\rho M_1^{\ 1} + a\omega_1 & 0 & \rho + a\rho_1 \\ -\rho_1 M_1^{\ 2} + 2M_1^{\ 1} & -\rho M_1^{\ 2} + (a+1)M_1^{\ 1} & 0 & -(a+1) \\ -\rho_1 M_1^{\ 3} + 3M_1^{\ 2} & -\rho M_1^{\ 3} + (a+2)M_1^{\ 2} & 0 & 0 \\ -\rho_1 M_1^{\ 4} + 4M_1^{\ 3} & -\rho M_1^{\ 4} + (a+3)M_1^{\ 3} & 0 & 0 \end{pmatrix}$$

The third column of C_1 is zero because M_a is independent of ρ_2. M_a' can be evaluated from $A_1M_a' = C_1$ by Gauss elimination method.

M_b' can be found by a similar method from the equation $A_2M_b' = C_2$, where:

$$C_2 = \begin{bmatrix} -\rho_2 M_2^{\;1} + \omega_2 & 0 & -\rho M_2^{\;1} + a\omega_2 & -\rho - a\rho_2 \\ -\rho_2 M_2^{\;2} + 2M_2^{\;1} & 0 & -\rho M_2^{\;2} + (a+1)M_2^{\;1} & a+1 \\ -\rho_2 M_2^{\;3} + 3M_2^{\;2} & 0 & -\rho M_2^{\;3} + (a+2)M_2^{\;2} & 0 \\ -\rho_2 M_2^{\;4} + 4M_2^{\;3} & 0 & -\rho M_2^{\;4} + (a+3)M_2^{\;3} & 0 \end{bmatrix}$$

The elements in the second column are zero because M_b is independent of ρ_1.